Electromagnetic signatures of strong-field gravity: light curves, spectra, and the polarized signal from black-hole accretion discs

Selected topics

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in collaboration with

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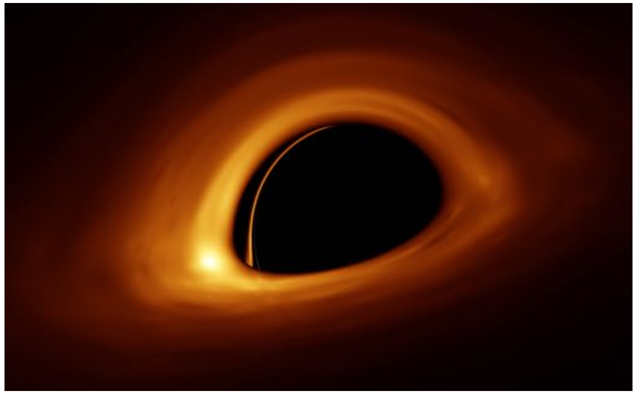

Objects and models

- Active galactic nuclei
- Stellar-mass black holes
- Intermediate-mass black holes (?)

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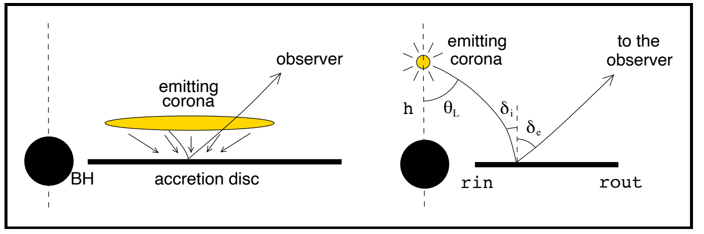

- Central black hole
- Accretion disc
  - ...geometrically thin, planar, non-self-gravitating
- Spectral features
  - ...time-dependent, non-axisymmetric

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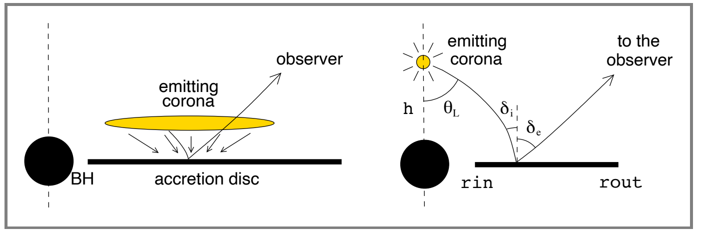
  - ...time-dependent, non-axisymmetric
- GR effects taken into account
- Link to a spectrum-fitting procedure (XSPEC)



GR lensing?

SCIENCE SERVICE

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2101 CONSTITUTION AVENUE
WASHINGTON, D.C.

Prof. Albert Einstein   Sept. 16, 1936
Institute for Advanced Study

Dear Prof. Einstein:

Last spring an apparently sincere laymen in science, Rudi Mandl, came into our offices here in the building of the National Academy of Sciences and dicussed a proposed test for the relativity theory based on observations during eclipses of stars.

We supplied Mr. Mandl with a small sum of money to enable him to visit you at Princeton and discuss it with you.

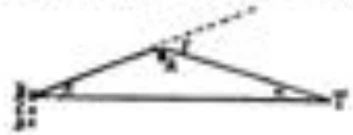
GR lensing?

Über eine mögliche Form fiktiver Doppelsterne. Von *O. Chwolson.*

Es ist gegenwärtig wohl als höchst wahrscheinlich anzunehmen, daß ein Lichtstrahl, der in der Nähe der Oberfläche eines Sternes vorbeigeht, eine Ablenkung erfährt. Ist γ diese Ablenkung und γ_0 der Maximumwert an der Oberfläche, so ist $\gamma_0 \geq \gamma \geq 0$. Die Größe des Winkels ist bei der Sonne $\gamma_0 = 1.''7$; es dürften aber wohl Sterne existieren, bei denen γ_0 gleich mehreren Bogensekunden ist; vielleicht auch noch mehr. Es sei A ein großer Stern (Gigant), T die Erde, B ein entfernter Stern; die Winkeldistanz zwischen A und B, von T aus gesehen, sei α, und der Winkel zwischen A und T, von B aus gesehen, sei β. Es ist dann

$$\gamma = \alpha + \beta .$$

Ist B sehr weit entfernt, so ist annähernd $\gamma = \alpha$. Es kann also α gleich mehreren Bogensekunden sein, und der Maximumwert von α wäre etwa gleich γ_0. Man sieht den Stern B von der Erde aus an zwei Stellen: direkt in der Richtung TB und außerdem nahe der Oberfläche von A, analog einem Spiegelbild. Haben wir mehrere Sterne B, C, D, so würden die Spiegelbilder umgekehrt gelegen sein wie in einem gewöhnlichen Spiegel, nämlich in der Reihenfolge D, C, B, wenn von A aus gerechnet wird (D wäre am nächsten zu A).



Der Stern A würde als fiktiver Doppelstern erscheinen. Teleskopisch wäre er selbstverständlich nicht zu trennen. Sein Spektrum bestände aus der Übereinanderlagerung zweier, vielleicht total verschiedenartiger Spektren. Nach der Interferenzmethode müßte er als Doppelstern erscheinen. Alle Sterne, die von der Erde aus gesehen rings um A in der Entfernung $\gamma_0 - \beta$ liegen, würden von dem Stern A gleichsam eingefangen werden. Sollte zufällig TAB eine gerade Linie sein, so würde, von der Erde aus gesehen, der Stern A von einem Ring umgeben erscheinen.

Ob der hier angegebene Fall eines fiktiven Doppelsternes auch wirklich vorkommt, kann ich nicht beurteilen.

Petrograd, 1924 Jan. 28. *O. Chwolson.*

Antwort auf eine Bemerkung von *W. Anderson.*

Daß ein Elektronengas einer Substanz mit negativem Brechungsvermögen optisch äquivalent sein müßte, kann bei dem heutigen Stand unserer Kenntnisse nicht zweifelhaft sein, da dasselbe einer Substanz von verschwindend kleiner Eigenfrequenz äquivalent ist.

Aus der Bewegungsgleichung

$$\varepsilon X = \mu\, d^2x/dt^2$$

eines Elektrons von der elektrischen Masse ε und der ponderabeln Masse μ folgt nämlich für einen sinusartig pendelnden Prozeß von der Frequenz ν die Gleichung

$$\varepsilon X = -(2\pi\nu)^2 \mu x .$$

Berücksichtigt man, daß εx das »Moment« eines schwingenden Elektrons ist, so erhält man für die Polarisation $p = n\varepsilon x$ eines Elektronengases mit n Elektronen pro Volumeinheit

$$p = -\varepsilon^2 n / [\mu (2\pi\nu)^2] \cdot X .$$

Hieraus folgt, daß die scheinbare Dielektrizitätskonstante

$$D = 1 + 4\pi p / X = 1 - \varepsilon^2 n / (\pi \mu \nu^2)$$

ist. $\sqrt{D}$ ist in diesem Falle der Brechungsexponent, also jedenfalls kleiner als 1. Es erübrigt sich bei dieser Sachlage, auf das Quantitative einzugehen.

Es sei noch bemerkt, daß ein Vergleich des Elektronengases mit einem Metall unstatthaft ist, weil die bei der elementaren Theorie der Metalle zugrundegelegte »Reibungskraft« bei freien Elektronen fehlt; das Verhalten der letzteren ist allein durch die Einwirkung des elektrischen Feldes und durch die Trägheit bedingt.

Berlin, 1924 April 15. *A. Einstein.*

O. Chwolson (1924), “Über eine mögliche Form fiktiver Doppelsterne”, Astronomische Nachrichten, 221, 329

GR lensing?

F. Link: positions and luminosities of the images formed by a gravitational lens (16 March 1936 session of the *Académie des Sciences de Paris*, Comptes Rendus 202, 917).

BAC Vol. 18 (1967), No. 4 215

PHOTOMETRICAL TABLES FOR EINSTEIN'S DEFLECTION OF THE LIGHT

F. Link, Astronomical Institute of the Czechoslovak Academy of Sciences, Praha

Received February 11, 1967

Abridged theory and numerical tables for photometrical and imaging action of Einstein's deflection are given in order to be used in stellar astronomy.

Фотометрические таблицы отклонения света Эйнштейна. Приводится сокращенная теория и нумерические таблицы для фотометрических и изображающих последствий отклонения света Эйнштейна в применении к звездной астрономии.

1. Introduction

The photometrical consequences of Einstein's deflection of light have been theoretical known since 1936 (Link 1936), but the observational test has not been performed up to the present time. Nevertheless, many authors have treated this problem again in the last few years bringing several interesting suggestions concerning the phenomena to be observed.

The common defect of the majority of these papers

or the total illumination

$$I = \frac{\varrho^2 + 2\varrho_0^2}{\varrho\sqrt{(\varrho^2 + 4\varrho_0^2)}}\,. \qquad (3)$$

The angle ϱ_0 called the critical distance is computed from

$$\varrho_0 = \sqrt{\left(k\,\frac{m}{m_\odot}\,\frac{a_\odot}{l_2}\,\frac{l_1}{l_1 + l_2}\right)}\,, \quad k = 1{\cdot}75''\,, \qquad (4)$$

Geometry of the model

Math of high-fr. elmg. waves

Basic equations – vacuum case: $F^{\mu\nu}{}_{;\nu} = 0, \qquad {}^{\star}F^{\mu\nu}{}_{;\nu} = 0.$

$$E^{\alpha} = F^{\alpha\beta}u_{\beta},\ {}^{\star}F_{\mu\nu} \equiv \tfrac{1}{2}\varepsilon_{\mu\nu}{}^{\rho\sigma}F_{\rho\sigma}$$

An electromagnetic wave is an approximate test-field solution of the Maxwell equations:

$$F_{\alpha\beta} = \Re e\left[u_{\alpha\beta}\, e^{\Im S(x)}\right].$$

A fixed background geometry is asssumed.

- Phase $S(x)$ ... rapidly varying function
- Amplitude $u_{\alpha\beta}$ ... slowly varying function
- Wave vector $k_{\alpha} \equiv S_{,\alpha}$ ... paralel transport, null geodesics

$$k_{\alpha;\beta}\,k^{\beta} = 0, \qquad k_{\alpha}k^{\alpha} = 0.$$

Polarization tensor

- Polarization tensor ... $J_{\alpha\beta\gamma\delta} \equiv \frac{1}{2}\langle F_{\alpha\beta}\bar{F}_{\gamma\delta}\rangle$
- In observer's rest-frame ... $J_{\alpha\beta} \equiv J_{\alpha\beta\gamma\delta}\, u^{\beta}\, u^{\delta} = \langle E_{\alpha}\bar{E}_{\beta}\rangle$
- Four observables $S_{_A}$ are obtained by projecting onto a tetrad, $e^{\alpha}_{(i)}$

"On the composition and resolution of streams of polarized light from different sources"

References: [1] Sir George Stokes (1852), Trans. Cambridge Phil. Soc., 9, 399

[2] Chandrasekhar (1950), *Radiative Transfer* (Oxford: Clarendon)

[3] Cocke & Holm (1972), Nature, 240, 161

[4] Jauch & Rohrlich (1955), *The Theory of Photons and Electrons* (Reading: Wesley)

Stokes parameters

$$S_0 \equiv J_{\alpha\beta}\left(e^{\alpha}_{(1)}e^{\beta}_{(1)} + e^{\alpha}_{(2)}e^{\beta}_{(2)}\right) = \langle |E_{(1)}|^2 + |E_{(2)}|^2\rangle$$

$$S_1 \equiv J_{\alpha\beta}\left(e^{\alpha}_{(1)}e^{\beta}_{(1)} - e^{\alpha}_{(2)}e^{\beta}_{(2)}\right) = \langle |E_{(1)}|^2 - |E_{(2)}|^2\rangle$$

$$S_2 \equiv J_{\alpha\beta}\left(e^{\alpha}_{(1)}e^{\beta}_{(2)} + e^{\alpha}_{(2)}e^{\beta}_{(1)}\right) = \langle E_{(1)}\bar{E}_{(2)} + E_{(2)}\bar{E}_{(1)}\rangle$$

$$S_3 \equiv \Im J_{\alpha\beta}\left(e^{\alpha}_{(1)}e^{\beta}_{(2)} - e^{\alpha}_{(2)}e^{\beta}_{(1)}\right) = \Im\langle E_{(1)}\bar{E}_{(2)} - E_{(2)}\bar{E}_{(1)}\rangle$$

S_1, S_2, and S_3 determine the polarization state.

References: [5] Anile (1989), *Relativistic fluids and magneto-fluids* (Cambridge)
[6] Madore (1974), Comm. Math. Phys., 38, 103
[7] Bičák & Hadrava (1975), A&A, 44, 389
[8] Breuer & Ehlers (1980), Proc. Roy. Soc. Lond. A, 370, 389
[9] Broderick & Blandford (2003), MNRAS, 342, 1280

Propagation law

Normalized Stokes parameters:

$$s_1 = S_1/S_0, \quad s_2 = S_2/S_0, \quad s_3 = S_3/S_0.$$

Degree of polarization:

$$\Pi_l = \sqrt{s_1^2 + s_2^2}, \quad \Pi_c = |s_3|, \quad \Pi = \sqrt{\Pi_l^2 + \Pi_c^2}.$$

Propagation through an arbitrary (empty) space-time:

$$\boxed{F_A \, dS_{\rm em} = F_A \, dS_{\rm obs}}$$

Five transfer functions

- The energy shift (gravitational and Doppler)
  - emitted photons are coming from places with high gravity
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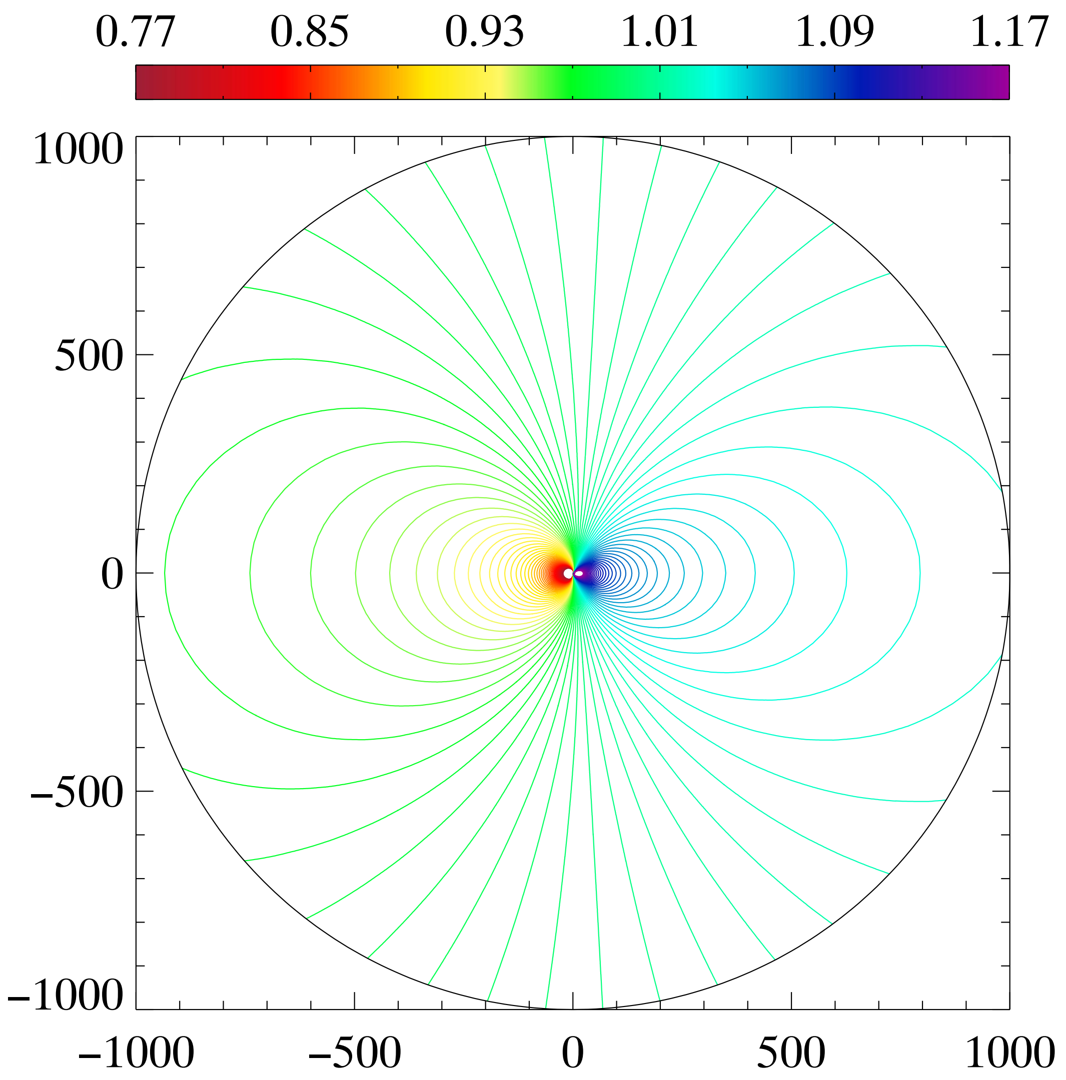
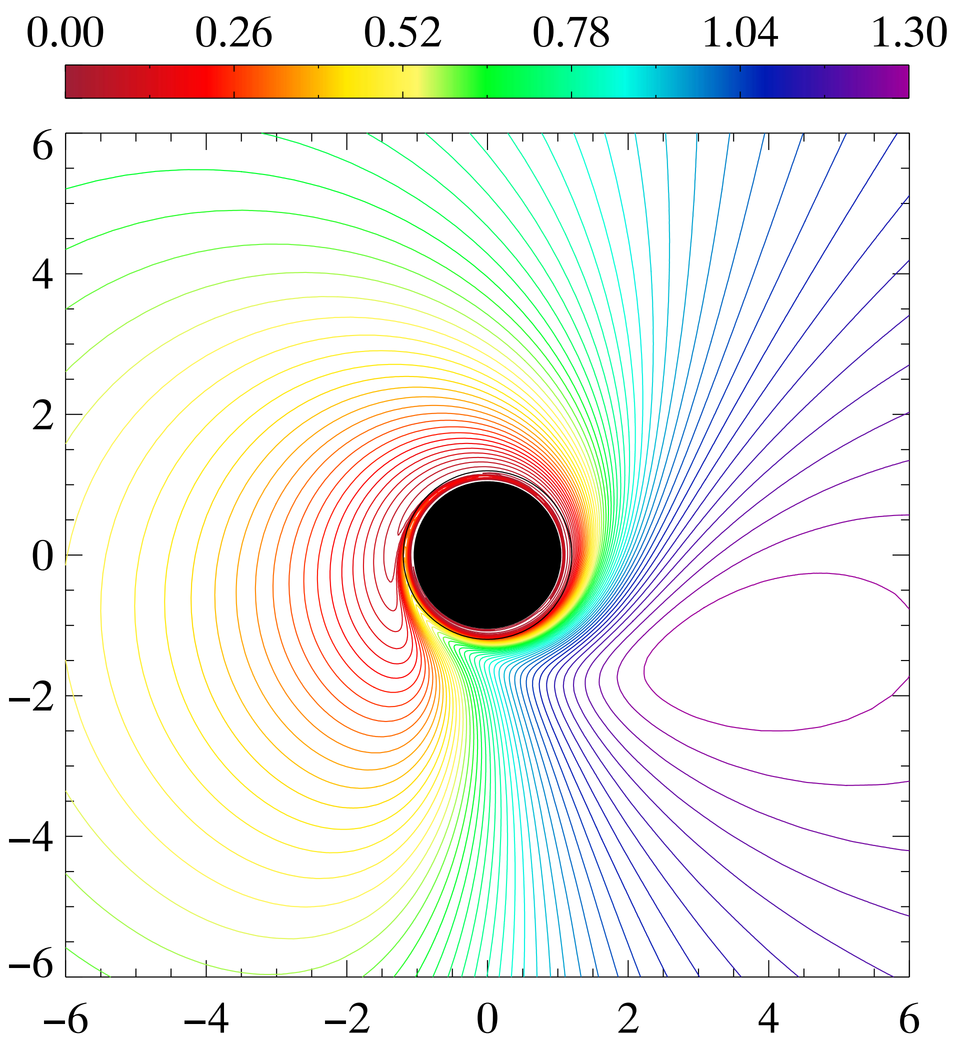
- The change of polarization angle
  - Polarization vector is parallel transported through gravitational field

The shift of photon energy, z



$a = M$, $\theta = 70°$ Dovčiak et al. (2004), ApJSS, 153, 205

The shift of photon energy, z

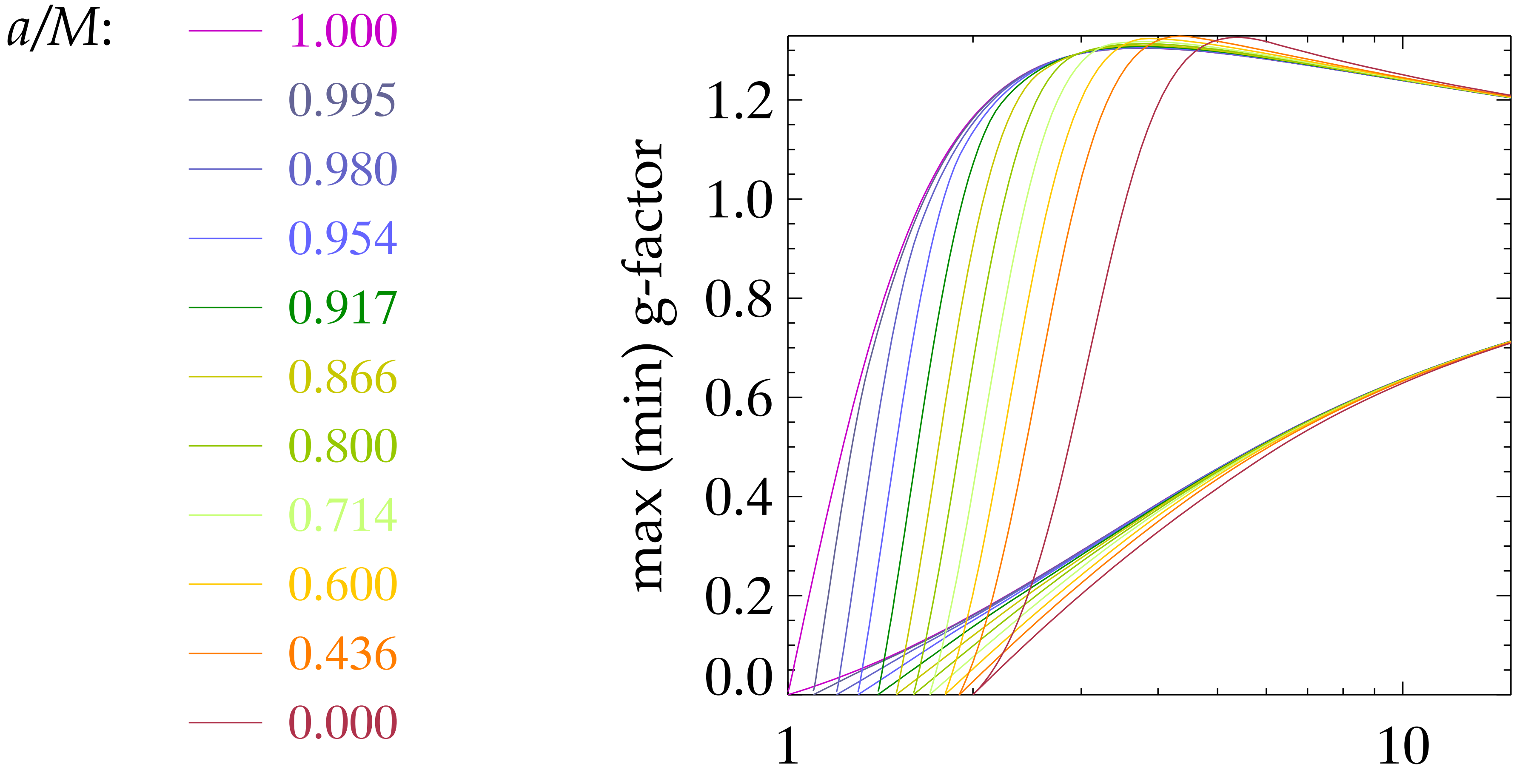


$a = M$, $\theta = 70^\circ$

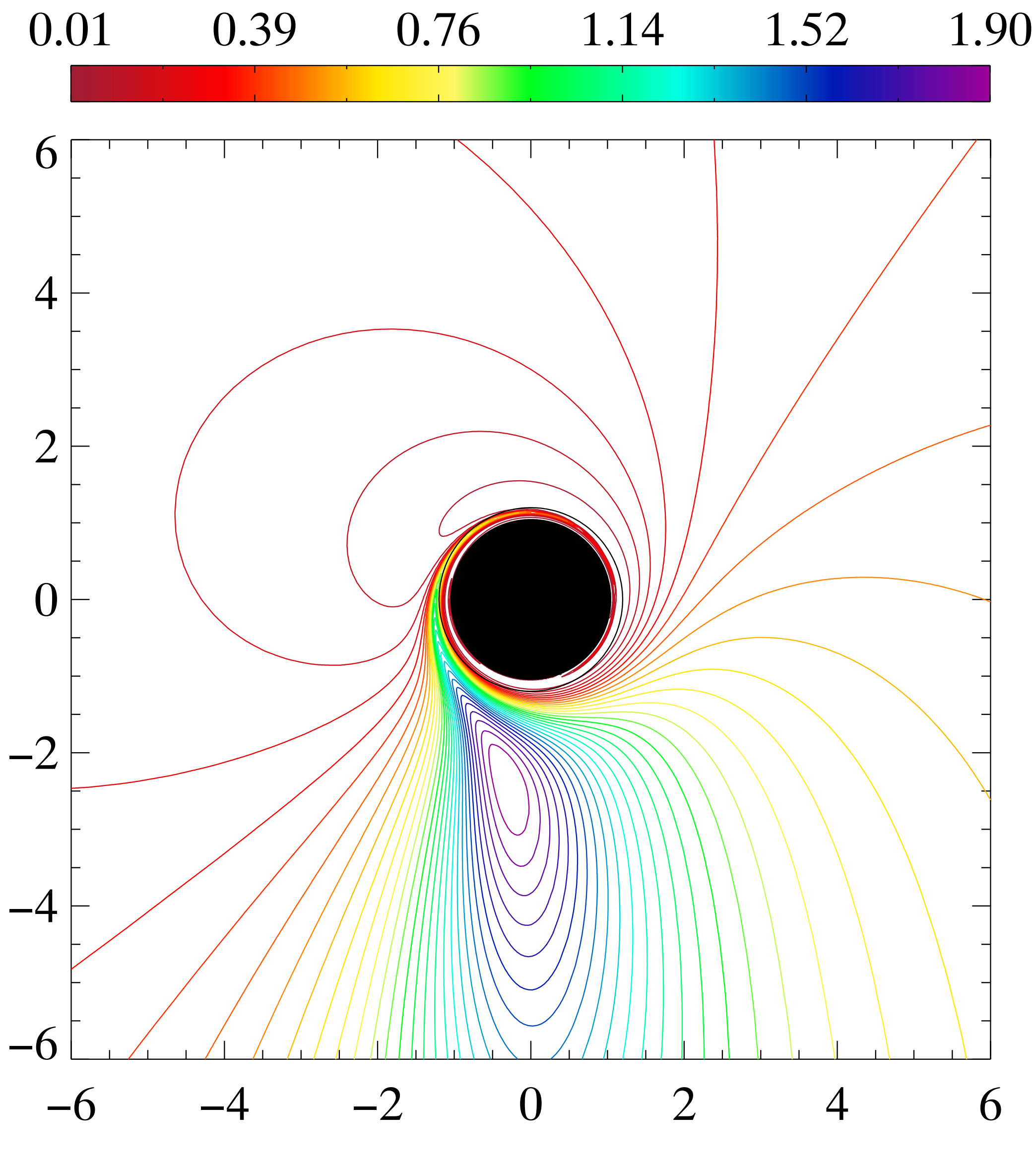
Dovčiak et al. (2004), ApJSS, 153, 205

The shift of photon energy, z

Maximum and minimum g-factor for θ_o=70°



Lensing effect, $S_{\rm em}/S_{\rm obs}$



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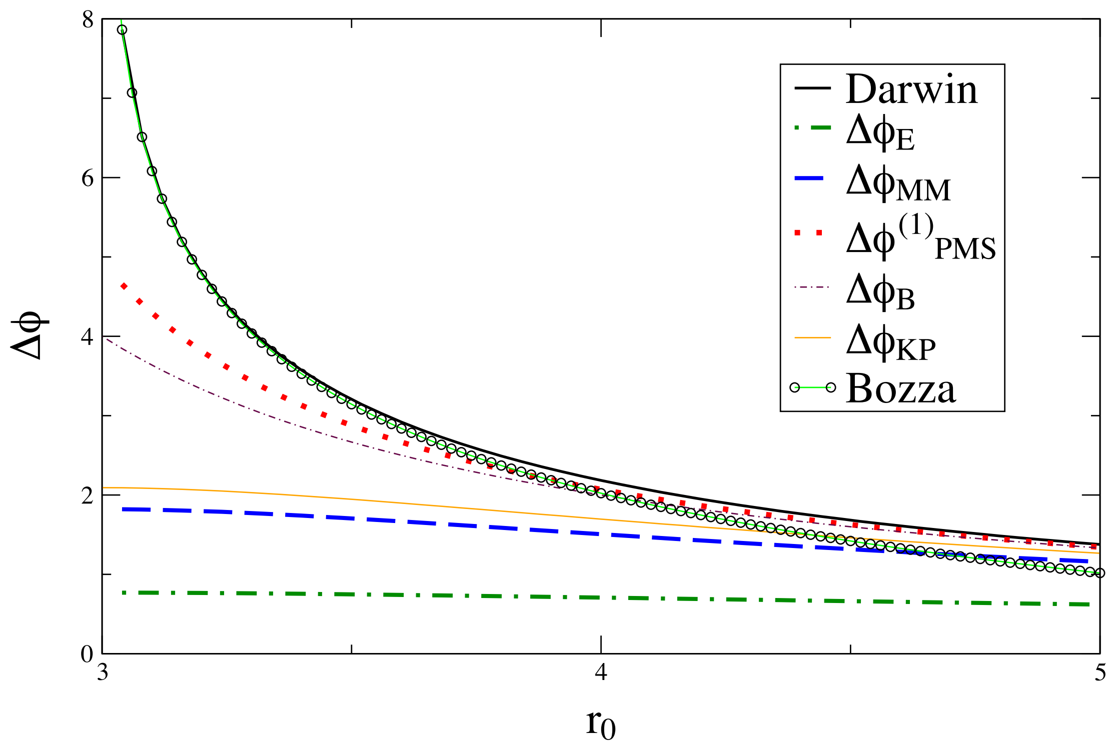


Darwin (1959)
Einstein (1911)
Mutka & Mähönen (2002)
Amore & Diaz (2006)
Beloborodov (2002)
Keeton & Peters (2005)
Bozza (2003)
Semerák (2015)

Light deflection and gravitational lensing: exact formula and analytical approximations.

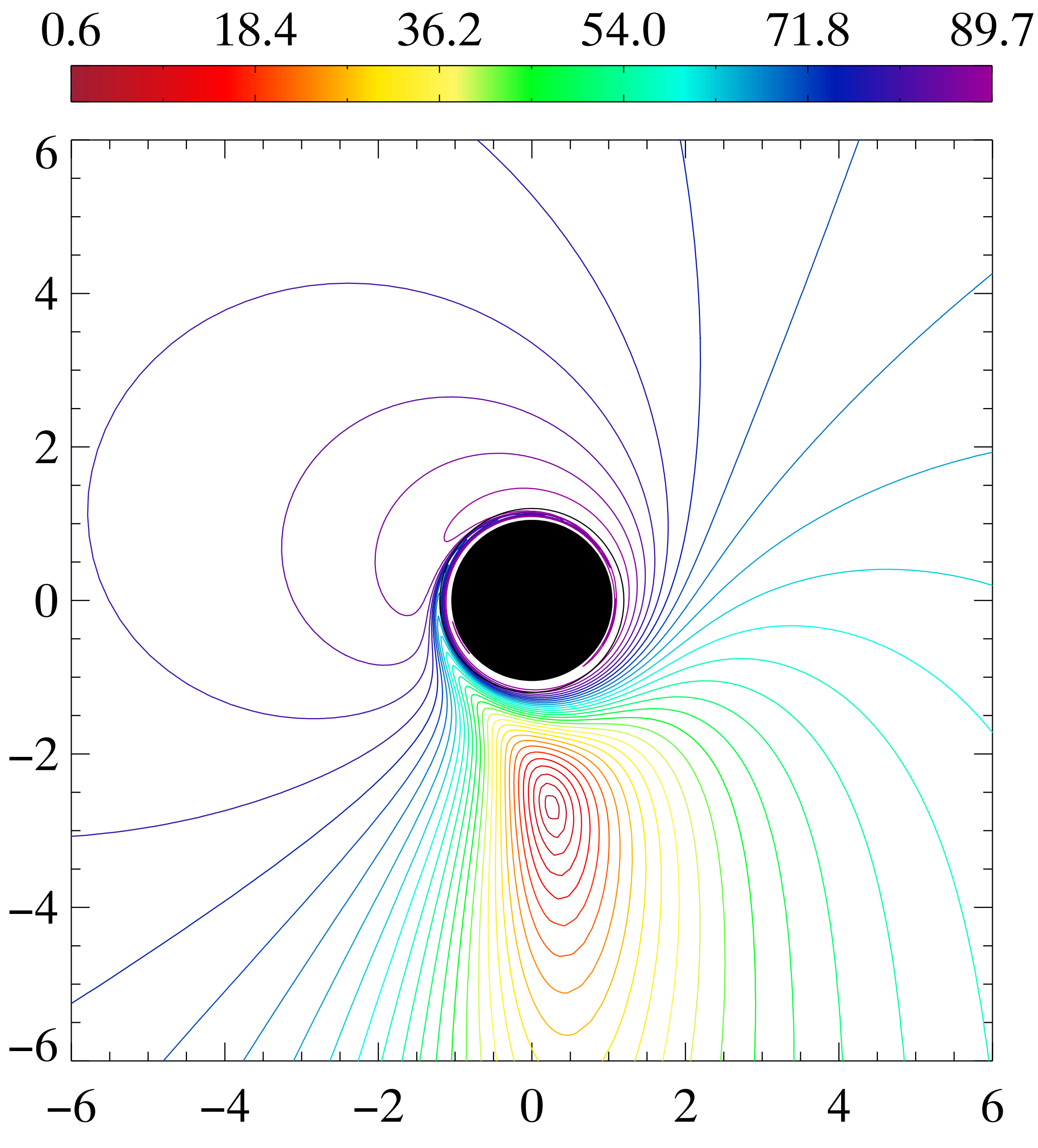
In Schwarzschild metric:

$$\delta\phi_D = 4\sqrt{r_0/(GM\Upsilon)}\left[F\left(\tfrac{\pi}{2},\kappa\right) - F\left(\varphi,\kappa\right)\right]$$

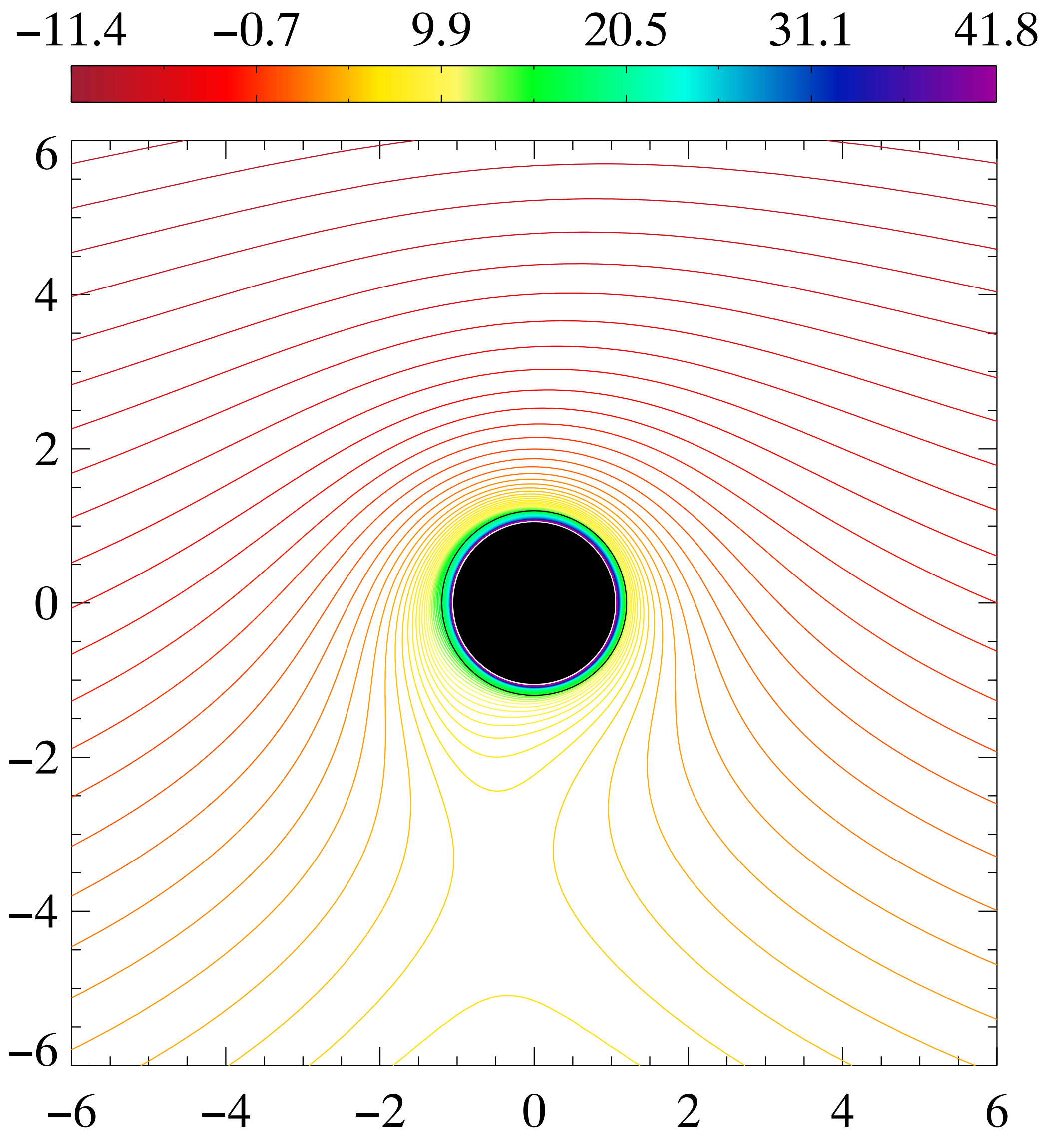
$$\approx \frac{4GM}{r_0} + \frac{7.78097G^2M^2}{r_0^2} + \frac{17.1047G^3M^3}{r_0^3} + \mathcal{O}\left[(GM/r_0)^4\right].$$

Amore & Diaz, Phys. Rev. D73 (2006) 083004

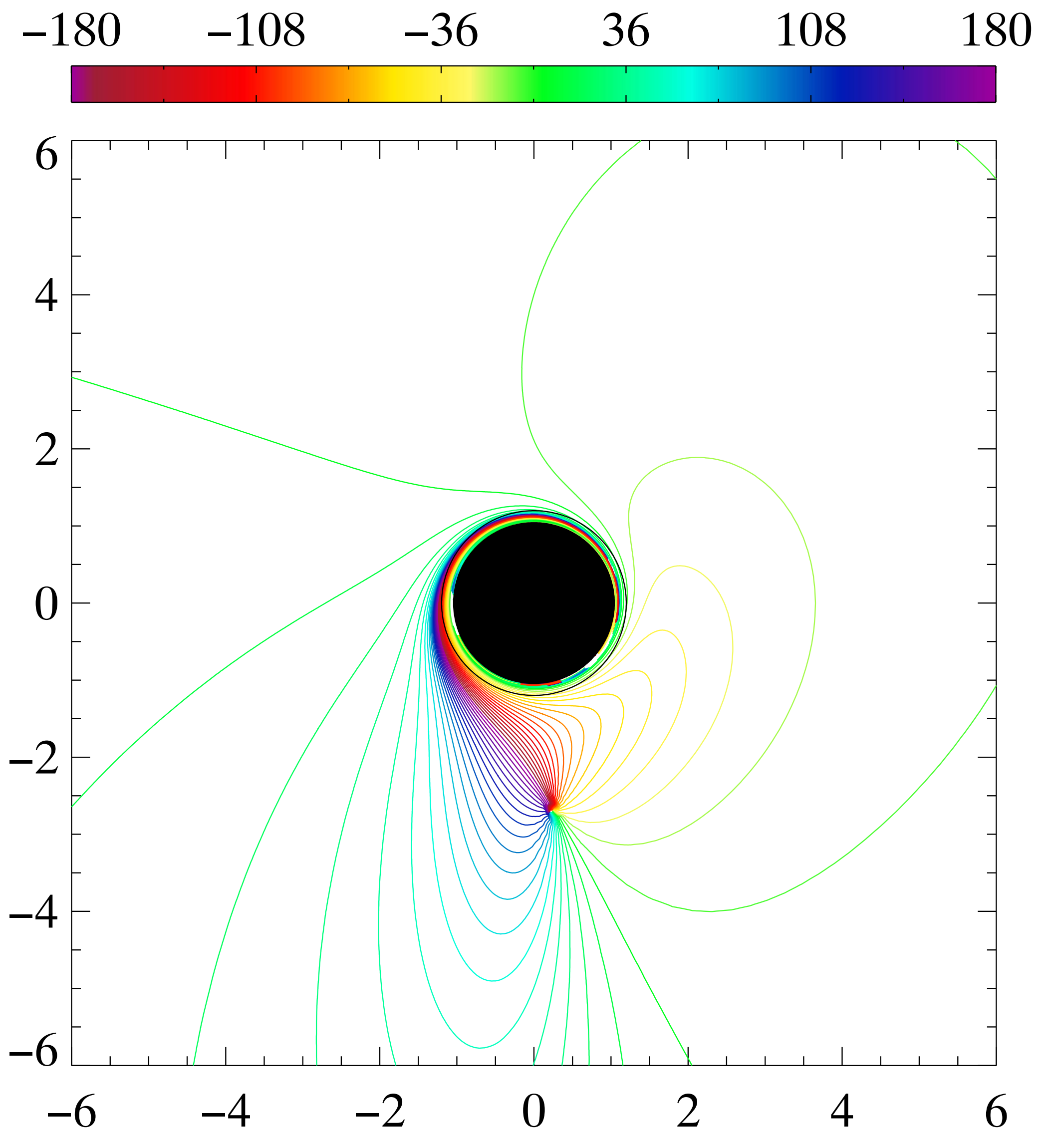
Emission angle, $\cos\delta_{em}$



Light-time effect, δt



Polarization angle, $\cos\psi$



Wave fronts in a BH spacetime

Schwarzschild metric,

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2.$$

Eikonal equation,

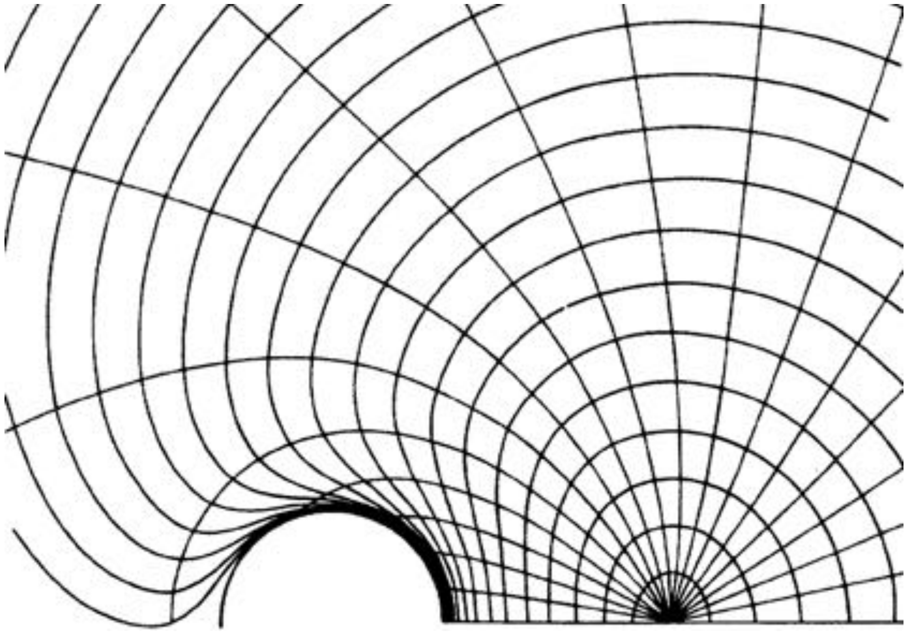
$$-\left(1 - \frac{2M}{r}\right) (\psi_{,r})^2 + \left(1 - \frac{2M}{r}\right)^{-1} (\psi_{,t})^2 - r^{-2}(\psi_{,\phi})^2 = 0.$$

Solved by separation of variables, $\psi(t, r, \phi) \equiv R(r) + \alpha\phi - \omega t$,

$$\left(1 - \frac{2M}{r}\right) (R')^2 = \left(1 - \frac{2M}{r}\right)^{-1} \omega^2 - r^{-2}\alpha^2.$$

Wave front: $\boxed{\psi(t_0 + n\,\delta t, r, \phi) = \psi(t_0, r_0, 0).}$

Wave fronts in a BH spacetime



Electromagnetic radiation does not influence geometry of the BH spacetime (to first order).

Wave fronts do not depend on polarization (in geometrical optics approximation).

The analogy:
light propagation in a vacuum curved spacetime versus material media in a flat spacetime.

The effective permeability: $\mu = \sqrt{1 - 2M/r}$.

Mashoon (1973); Hanni (1977); . . .

Wave fronts in a BH spacetime

Kerr metric,

$$ds^2 = -\frac{\Delta}{\Sigma}\Big(dt - a\sin^2\theta\, d\phi\Big)^2 + \frac{\Sigma}{\Delta}\, dr^2 + \Sigma\, d\theta^2$$

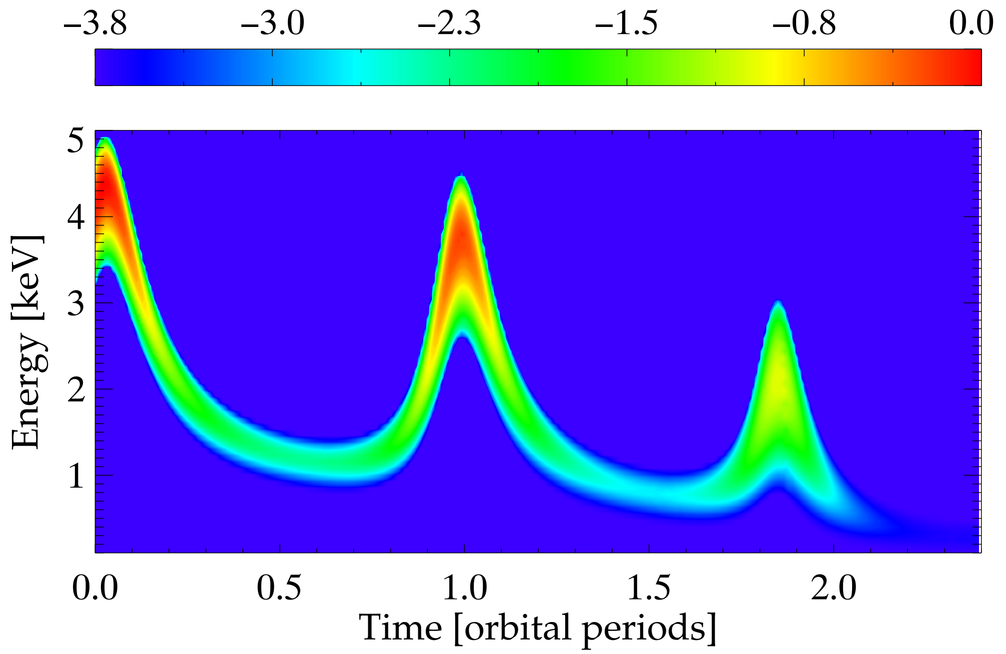
$$+\frac{\sin^2\theta}{\Sigma}\Big[a\, dt - (r^2 + a^2)\, d\phi\Big]^2.$$

The separation of variables and solution for the eikonal equation follow from Carter's solution of the scalar wave equation,

$$\psi = R(r) + T(\theta) + \alpha\phi - \omega t.$$

Wave fronts exhibit the frame dragging effect.

Example 1: Orbiting spot



Reviews:

- Fabian, Iwasawa, Reynolds, Young, (2000), "Broad Iron Lines in Active Galactic Nuclei", PASP, 112, 1145
- Reynolds & Nowak (2003), "X-rays from active galactic nuclei: relativistically broadened emission lines", Phys. Rep., 337, 389
- Karas (2006), "Theoretical aspects of relativistic spectral features", Astronomische Nachrichten, 327, 961

Example 2: Polarization

Thermal emission from an accretion disc can be polarized due to scattering in the disc atmosphere.

GR changes the observed polarization at infinity: rotation of the polarization angle.

We compute the polarization degree and the angle as functions of

- energy (~ 2–10 keV),
- view angle of the observer,
- spin of the black hole,
- optical thickness of the atmosphere.

Connors, Stark, & Piran (1980), ApJ, 235, 224

Dovčiak, Muleri, Goosmann, Karas, & Matt (2008), MNRAS, 391, 32

Li-Xin Li, Narayan, & McClintock (2009), ApJ, 691, 847

Thank you!

For further details:

V. Karas et al. (2019), in “Radiative Signatures from the Cosmos”, ASP Conference Series, Vol. 519, Proceedings of a conference held at Sorbonne University, Paris, France. Eds. K. Werner, C. Stehle, T. Rauch, & T. Lanz. San Francisco: Astronomical Society of the Pacific, p.293 (arXiv:1901.06496)

V. Karas (2006), “Theoretical aspects of relativistic spectral features”, Astronomische Nachrichten, Vol. 327, p.961 (arXiv:astro-ph/0609645)

S. Britzen et al. (2019), “A cosmic collider: Was the IceCube neutrino generated in a precessing jet-jet interaction in TXS 0506+056?”, Astronomy & Astrophysics, Vol. 630, id.A103