

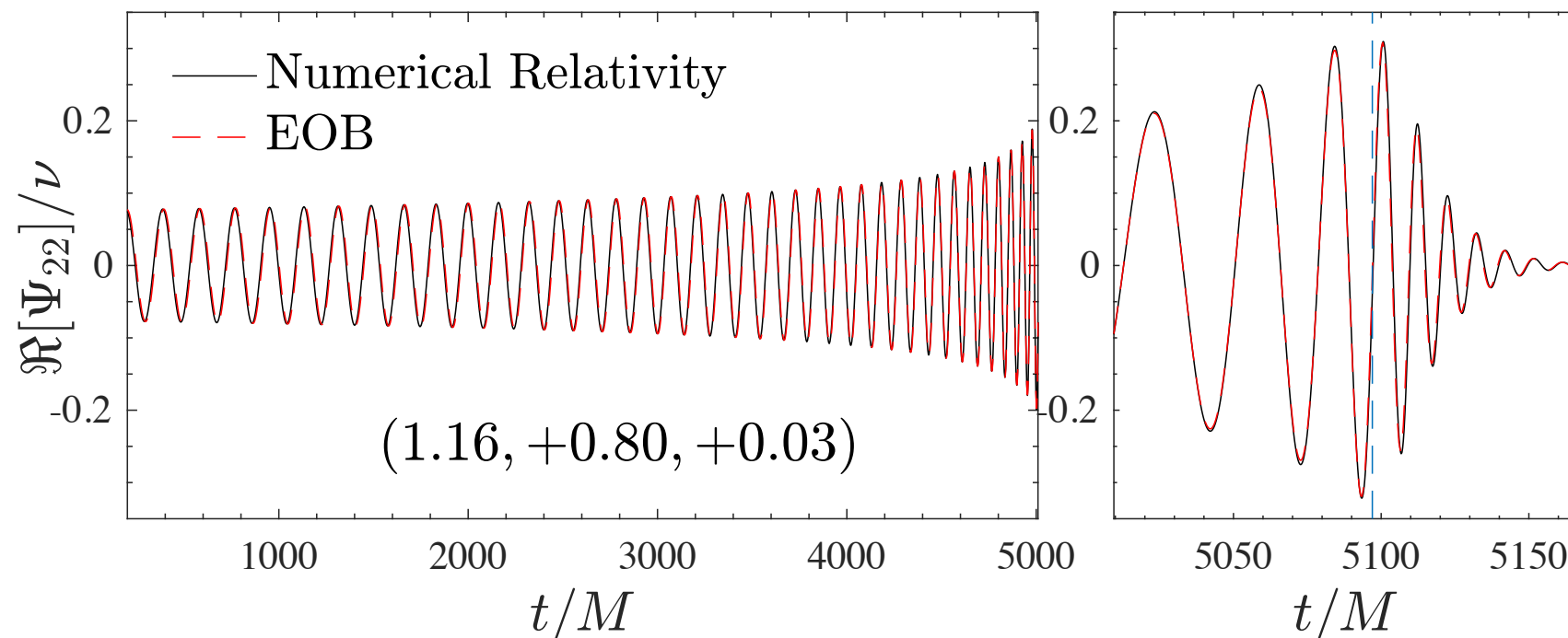
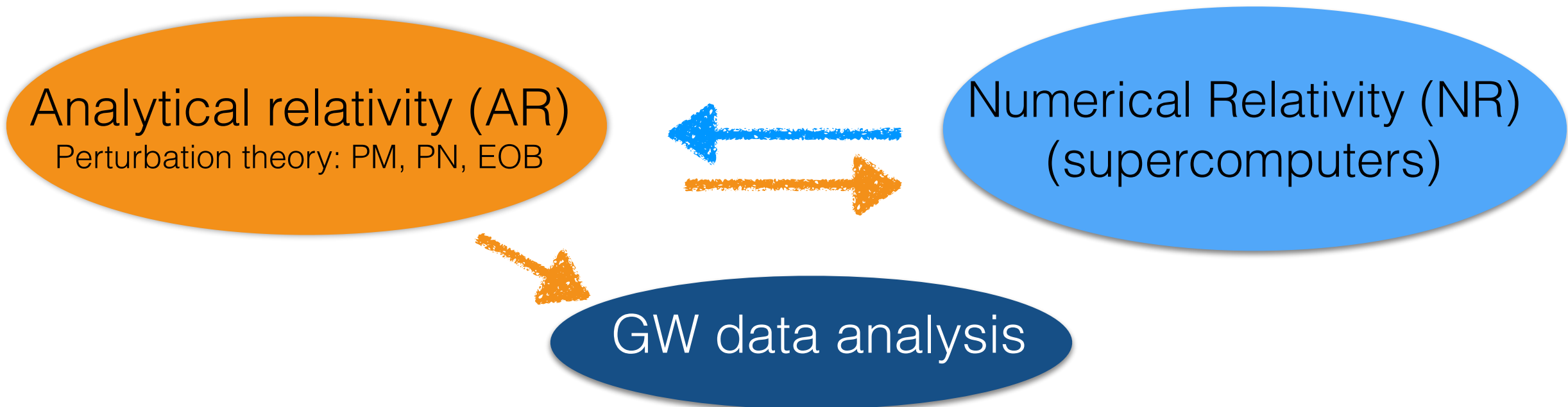
Coalescing compact binaries: the theory

Alessandro Nagar
INFN Torino and IHES

with: [S. Bernuzzi](#), [S. Akcay](#), [M. Agathos](#), [M. Breschi](#), T. Damour,
R. Gamba, P. Rettegno, G. Riemenschneider, F. Martinetti, D. Chiaramello

THEORY for CBC

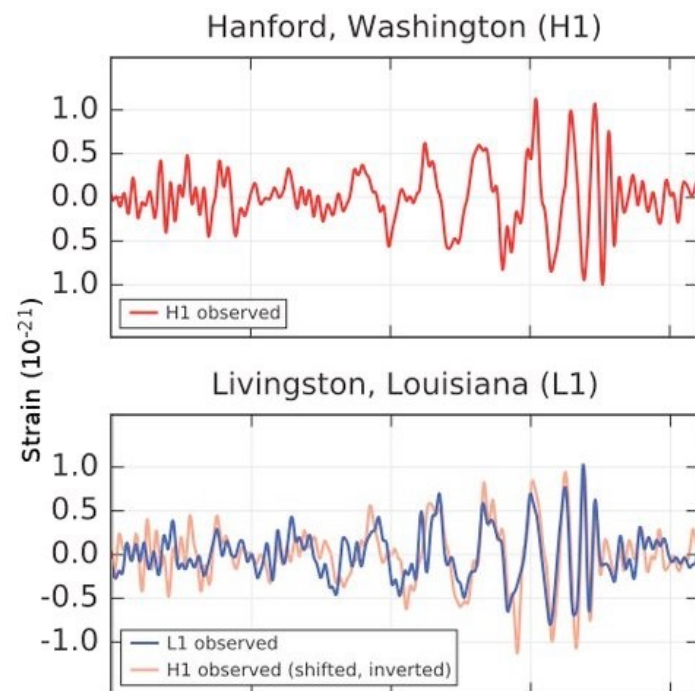
- Interface between Analytical & Numerical Relativity for GW data-analysis
- 2-body problem in General Relativity



Challenges:

- physical completeness
- accuracy
- efficiency (AR vs NR)
- 10^7 templates needed for a single event

Why waveform templates?



$$\text{strain} = \frac{\delta L}{L}$$

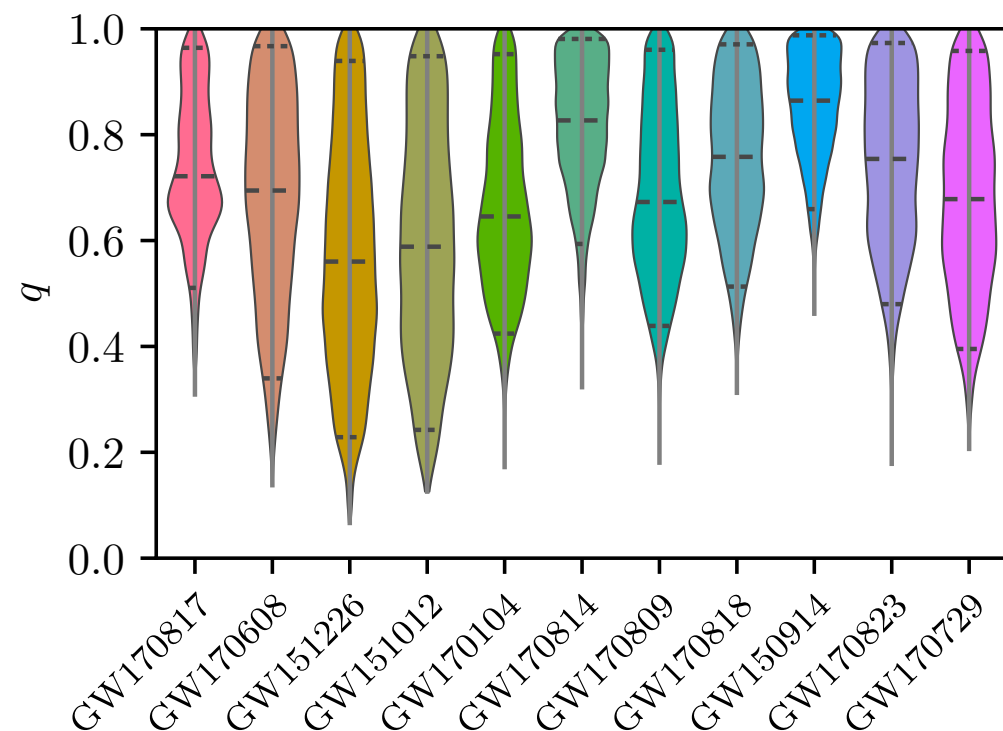
Symmetric mass ratio

$$\nu \equiv \frac{m_1 m_2}{(m_1 + m_2)^2} = 0.2466$$

GW150914 parameters:

$$\begin{aligned} m_1 &= 35.7 M_\odot \\ m_2 &= 29.1 M_\odot \\ M_f &= 61.8 M_\odot \\ a_1 &\equiv S_1/(m_1^2) = 0.31^{+0.48}_{-0.28} \\ a_2 &\equiv S_2/(m_2^2) = 0.46^{+0.48}_{-0.42} \\ a_f &\equiv \frac{J_f}{M_f^2} = 0.67 \\ q &\equiv \frac{m_1}{m_2} = 1.27 \end{aligned}$$

O2 events: GWTC-1: arXiv:1811.12907



Matched filtering: detection and parameter estimation

$$\langle \text{output} | h_{\text{template}} \rangle = \int \frac{df}{S_n(f)} o(f) h_{\text{template}}^*(f)$$

Analytical formalism:
theoretical understanding
of the coalescence process

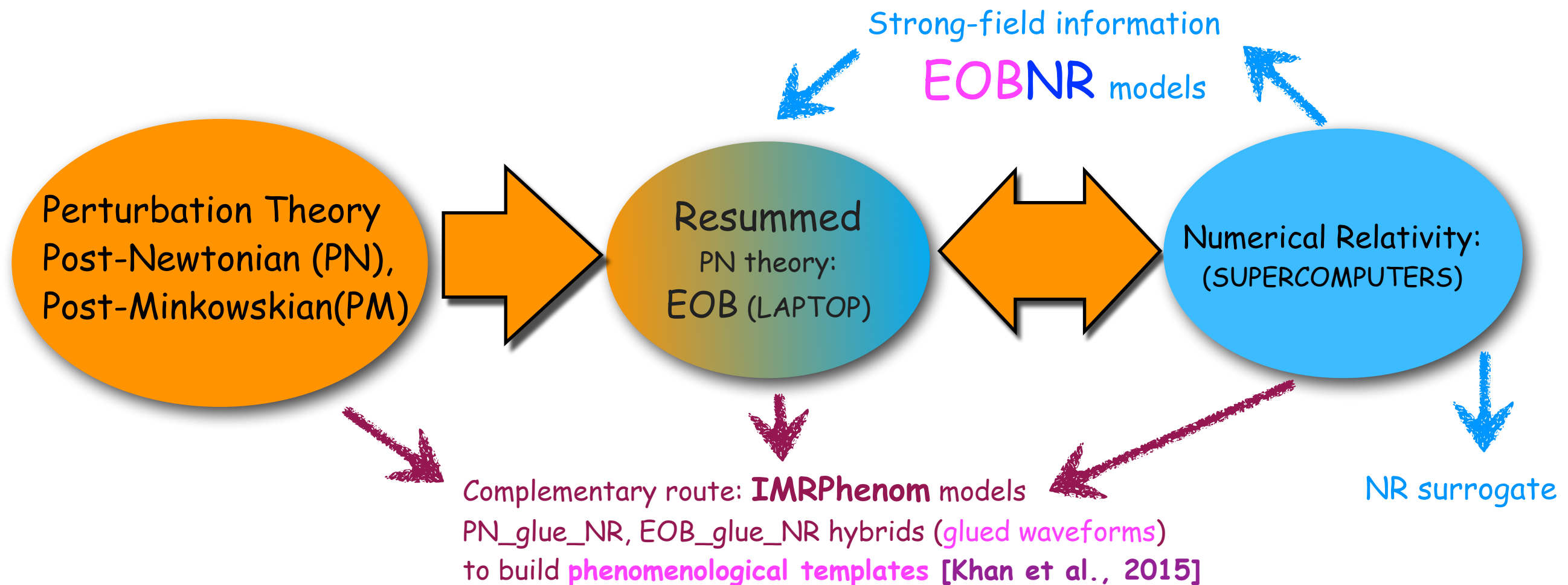
2-body problem in GR

Hamiltonian: conservative part of the dynamics

Radiation reaction: mechanical energy/angular momentum goes away in GWs and backreacts on the system.

The (closed) **orbit CIRCULARIZES** and **SHRINKS** with time

Waveform



EFFECTIVE-ONE-BODY (EOB)

approach to the general relativistic two-body problem

(Buonanno-Damour 99, 00, Damour-Jaranowski-Schäfer 00, Damour 01, Damour-Nagar 07, Damour-Iyer-Nagar 08)

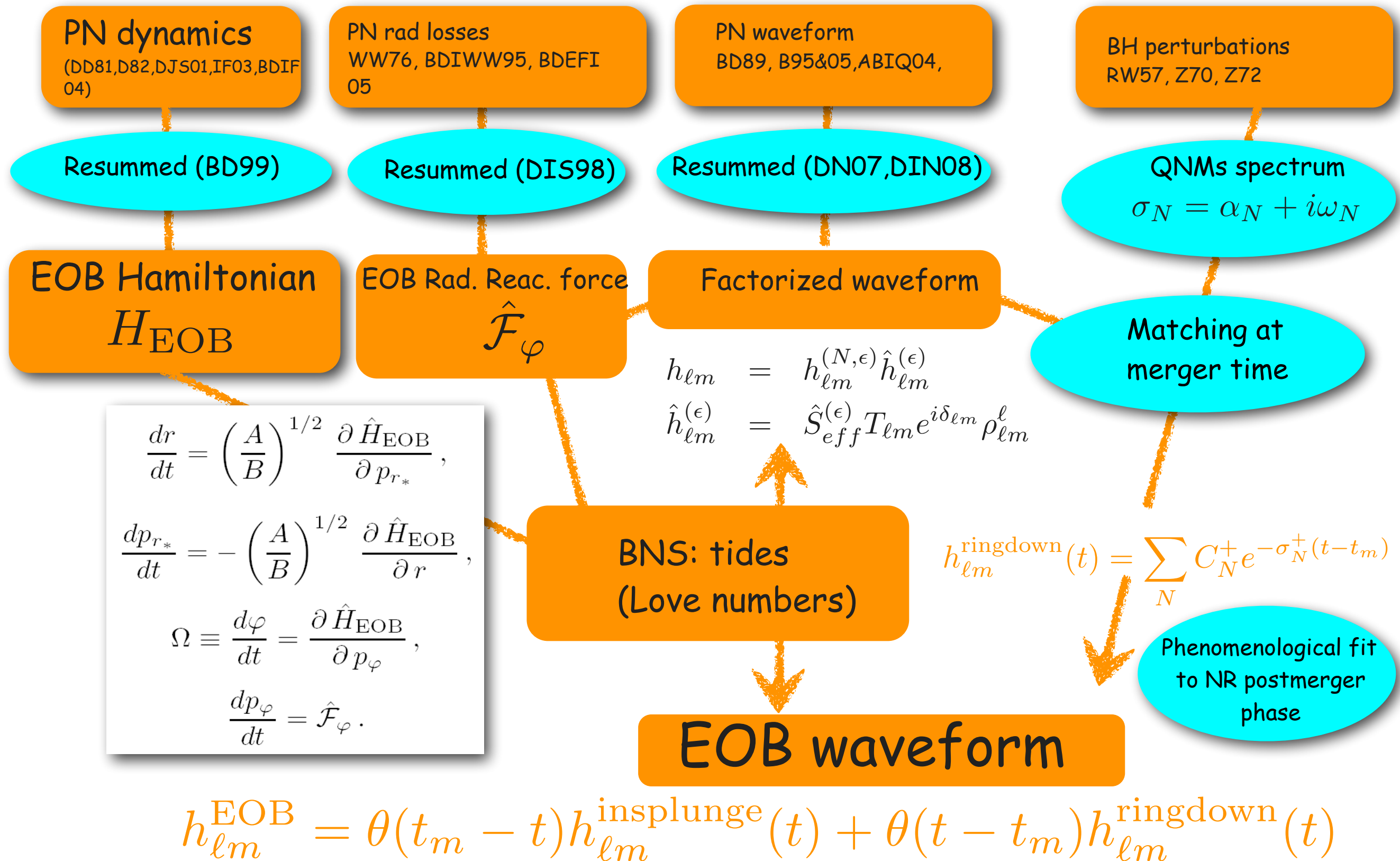
key ideas:

- (1) Replace two-body dynamics (m_1, m_2) by dynamics of a particle ($\mu \equiv m_1 m_2 / (m_1 + m_2)$) in an effective metric $g_{\mu\nu}^{\text{eff}}(u)$, with

$$u \equiv GM/c^2 R, \quad M \equiv m_1 + m_2$$

- (2) Systematically use **RESUMMATION** of PN expressions (both $g_{\mu\nu}^{\text{eff}}$ and \mathcal{F}_{RR}) based on various physical requirements
- (3) Require **continuous deformation w.r.t.**
 $\nu \equiv \mu/M \equiv m_1 m_2 / (m_1 + m_2)^2$ in the interval $0 \leq \nu \leq \frac{1}{4}$

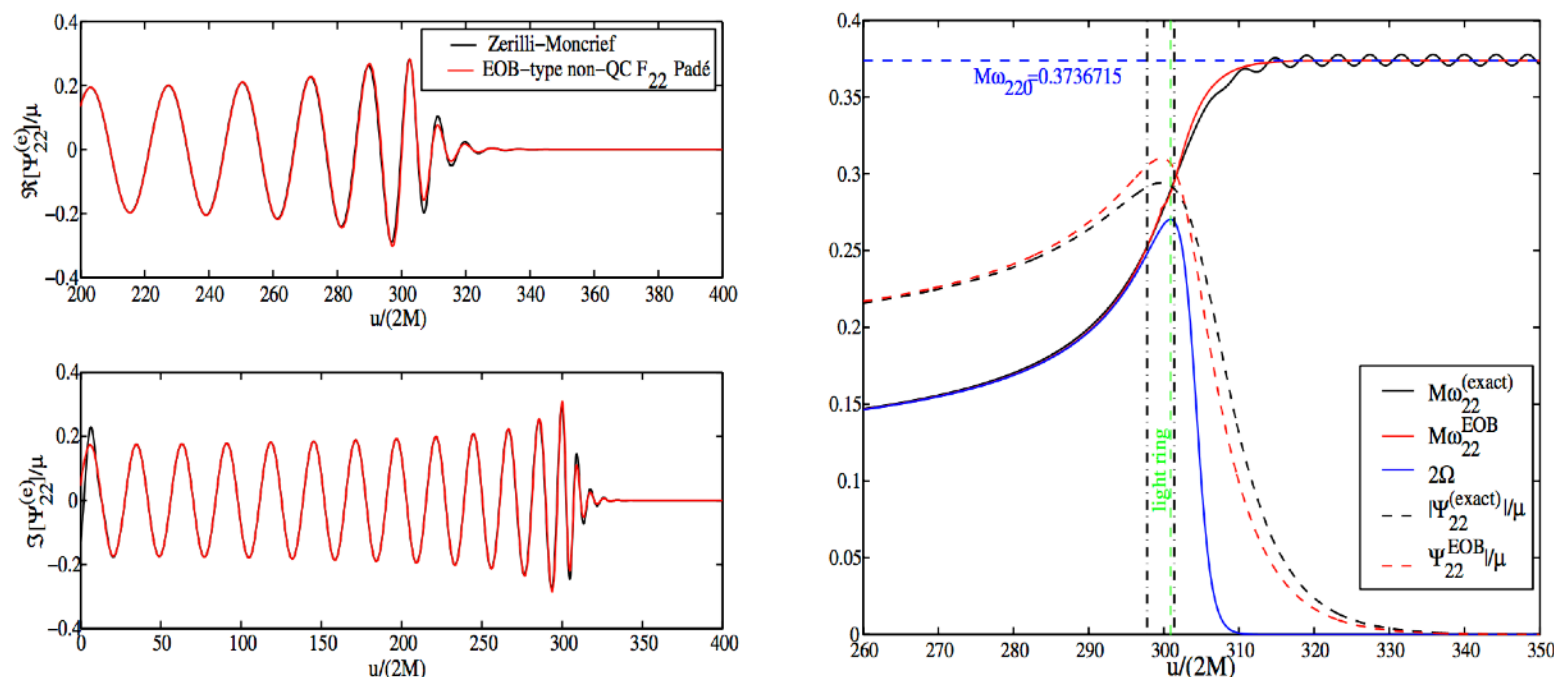
STRUCTURE OF THE EOB FORMALISM



+ GSF + EOB based on Post-Minkowskian approximation

Extreme-mass-ratio limit (2007)

- Laboratory to learn each physical element entering the coalescence
- Accurate waveform computation using Regge-Wheeler-Zerilli (Schwarzschild) or Teukolsky (Kerr) perturbation equations



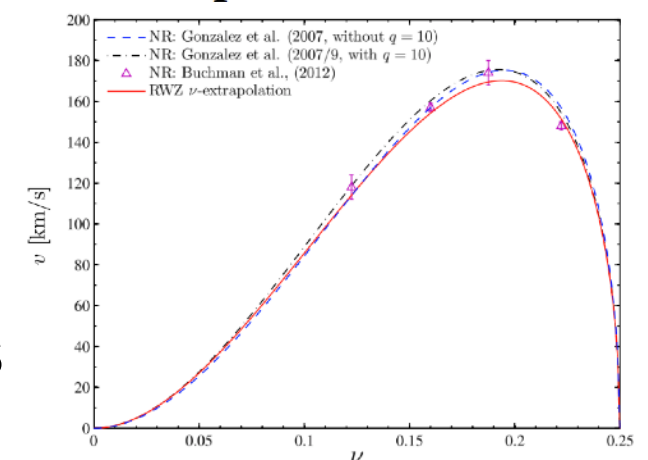
- Several aspects of the phenomenon explored in detail
- Several papers with Bernuzzi+ (multipoles, GW-recoil, spin etc.): [Teukode](#)

PHYSICAL REVIEW D **88**, 121501(R) (2013)

Gravitational recoil in nonspinning black-hole binaries: The span of test-mass results

Alessandro Nagar

RWZ-extrapolated final recoil velocity and “full” NR calculations



EOBNR Models

AEI (Ligo): LAL

SEOBNRv4 (spin-aligned)

SEOBNRv4_HM (spin-aligned, 22,21,33,44,55 modes)

SEOBNRv4P_HM (precessing spins, 22,21,33,44,55 modes)

SEOBNRv4T (tides)

(**Virgo**): standalone C & LAL implementation

TEOBResumS (spin-aligned, tides, BBH, BNS, BHNS)

TEOBiResumS (higher modes, in progress)

Differences:

- Hamiltonian (gauge + spin sector. Spin-spin)
- Resummation of the interaction potential
- Radiation reaction
- Effective representation of merger and post merger
- ESSENTIALLY: different deformation of the test-mass limit

EOB Hamiltonian

EOB Hamiltonian

$$H_{\text{EOB}} = M \sqrt{1 + 2\nu \left(\hat{H}_{\text{eff}} - 1 \right)}$$

All functions are a **-dependent deformation** of the Schwarzschild ones

$$\underline{A(r) = 1 - 2u + 2\nu u^3 + a_4 \nu u^4}$$

$$a_4 = \frac{94}{3} - \frac{41}{32} \pi^2 \simeq 18.6879027$$

$$A(r)B(r) = 1 - 6\nu u^2 + 2(3\nu - 26)\nu u^3$$

$$u = GM/(c^2 R)$$

Simple effective Hamiltonian:

$$\hat{H}_{\text{eff}} \equiv \sqrt{p_{r_*}^2 + \underset{\substack{\uparrow \\ \text{Crucial EOB radial potential}}}{A(r)} \left(1 + \frac{p_\varphi^2}{r^2} + \underset{\substack{\uparrow \\ \text{Contribution at 3PN}}}{z_3} \frac{p_{r_*}^4}{r^2} \right)} \quad p_{r_*} = \left(\frac{A}{B} \right)^{1/2} p_r$$

Crucial EOB radial potential

Contribution at 3PN

TEOBResumS - I

4PN analytically complete + 5PN logarithmic term in the $A(u)$ function:

[Damour 2009, Blanchet et al. 2010, Barack, Damour & Sago 2010, Le Tiec et al. 2011, Barausse et al. 2011, Akcay et al. 2012, Bini & Damour 2013, Damour Jaranowski & Schaefer 2014].

$$A_{5\text{PN}}^{\text{Taylor}} = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2 \right) \nu u^4 + \underbrace{\nu [a_5^c(\nu) + a_5^{\text{ln}} \ln u]}_{4\text{PN}} u^5 + \underbrace{\nu [a_6^c(\nu) + a_6^{\text{ln}} \ln u]}_{5\text{PN}} u^6$$

$$\left. \begin{aligned} a_5^{\text{log}} &= \frac{64}{5} && 1\text{PN} && 2\text{PN} && 3\text{PN} \\ a_5^c &= a_{5_0}^c + \nu a_{5_1}^c \\ a_{5_0}^c &= -\frac{4237}{60} + \frac{2275}{512}\pi^2 + \frac{256}{5}\log(2) + \frac{128}{5}\gamma \\ a_{5_1}^c &= -\frac{221}{6} + \frac{41}{32}\pi^2 \\ a_6^{\text{log}} &= -\frac{7004}{105} - \frac{144}{5}\nu \end{aligned} \right\} \begin{array}{l} 4\text{PN fully known ANALYTICALLY!} \\ 5\text{PN logarithmic term (analytically known)} \end{array}$$

NEED ONE "effective" 5PN parameter from NR waveform data: $a_6^c(\nu)$

State-of-the-art EOB potential (5PN-resummed):

$$A(u; \nu, a_6^c) = P_5^1 [A_{5\text{PN}}^{\text{Taylor}}(u; \nu, a_6^c)]$$

TEOBResumS - II

Resummation of the waveform (and flux) multipole by multipole (**CRUCIAL!**)

[Damour&Nagar 2007, Damour, Iyer, Nagar 2008]

Next-to-quasi-circular correction

$$h_{\ell m} \equiv \underbrace{h_{\ell m}^{(N, \epsilon)}}_{\text{Newtonian}} \underbrace{\hat{h}_{\ell m}^{(\epsilon)}}_{\text{PN-correction}} \underbrace{\hat{h}_{\ell m}^{\text{NQC}}}_{\text{NQC}}$$

Newtonian \times PN \times NQC

$$\hat{h}_{\ell m}^{(\epsilon)} = \hat{S}_{\text{eff}}^{(\epsilon)} T_{\ell m} e^{i\delta_{\ell m}} \rho_{\ell m}^{\ell}$$

Remnant phase and modulus corrections: "improved" PN series

The "Tail factor"

$$T_{\ell m} = \frac{\Gamma(\ell + 1 - 2i\hat{k})}{\Gamma(\ell + 1)} e^{\pi\hat{k}} e^{2i\hat{k} \ln(2kr_0)}$$

Effective source:

EOB (effective) energy (even-parity modes)

EOB angular momentum (odd-parity modes)

Resums an infinite number of leading logarithms in tail effects (hereditary contributions)

TEOBResumS - III

Damour&AN 2014: NR-based phenomenological description of postmerger phase

Factorize the fundamental

QNM, fit what remains

$$h(\tau) = e^{\sigma_1 \tau - i\phi_0} \bar{h}(\tau)$$

$$\bar{h}(\tau) \equiv A_{\bar{h}} e^{i\phi_{\bar{h}}(\tau)}.$$

$$A_{\bar{h}}(\tau) = c_1^A \tanh(c_2^A \tau + c_3^A) + c_4^A,$$

$$\phi_{\bar{h}}(\tau) = -c_1^\phi \ln \left(\frac{1 + c_3^\phi e^{-c_2^\phi \tau} + c_4^\phi e^{-2c_2^\phi \tau}}{1 + c_3^\phi + c_4^\phi} \right)$$

$$c_2^A = \frac{1}{2} \alpha_{21},$$

$$\alpha_{21} = \alpha_2 - \alpha_1$$

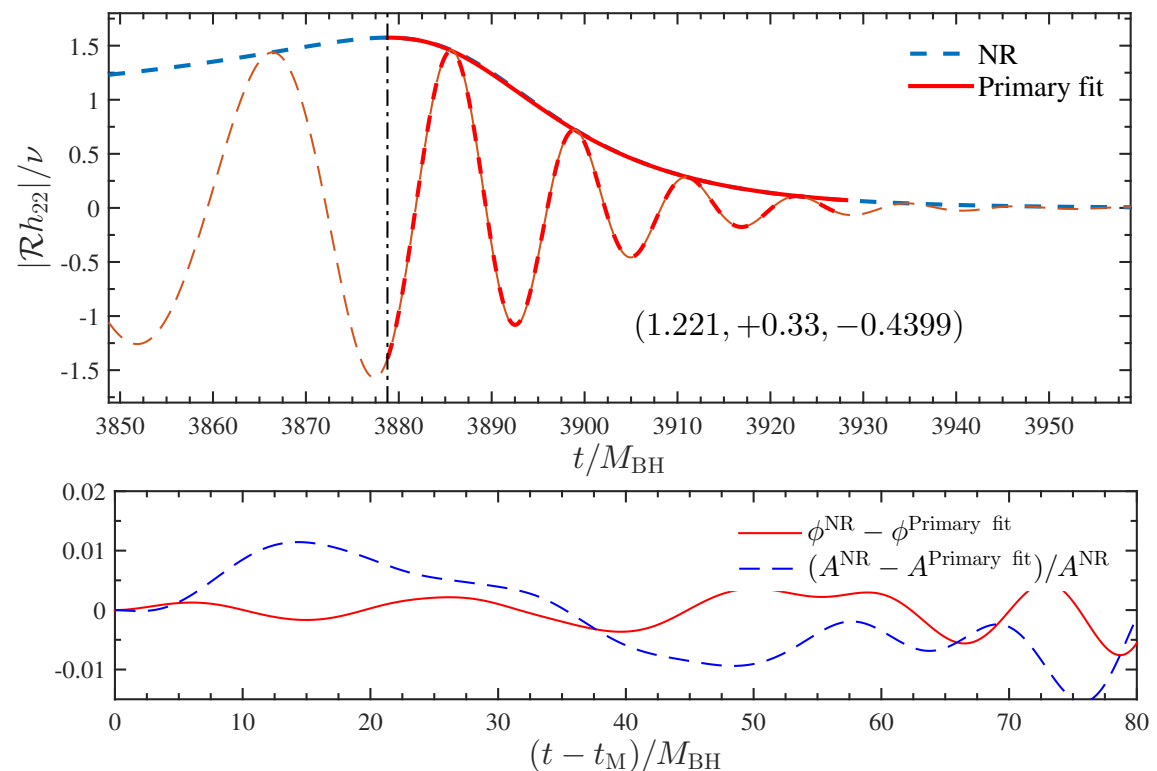
$$c_4^A = \hat{A}_{22}^{\text{mrg}} - c_1^A \tanh(c_3^A),$$

$$c_1^A = \hat{A}_{22}^{\text{mrg}} \alpha_1 \frac{\cosh^2(c_3^A)}{c_2^A},$$

$$c_1^\phi = \Delta\omega \frac{1 + c_3^\phi + c_4^\phi}{c_2^\phi (c_3^\phi + 2c_4^\phi)},$$

$$\Delta\omega \equiv \omega_1 - M_{\text{BH}} \omega_{22}^{\text{mrg}}$$

$$c_2^\phi = \alpha_{21},$$



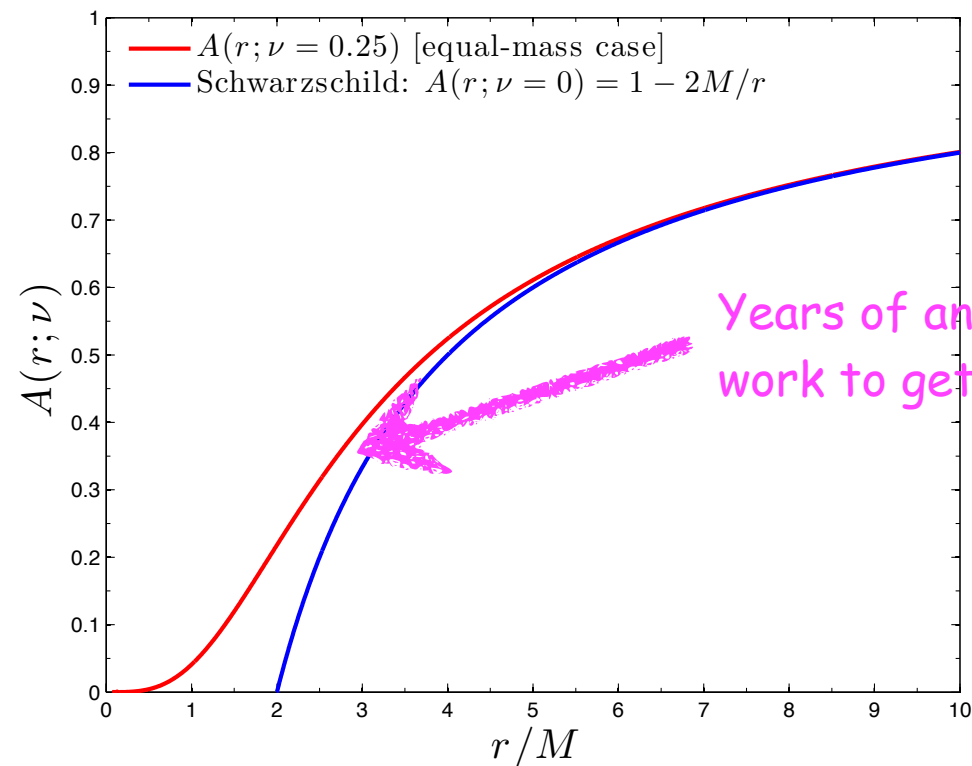
Good performance of primary fits (modulo details...)

Do this for various SXS dataset and then build up a (simple-minded) interpolating fit

Black-list:

- (1) the structure due to $m < 0$ modes is not included (yet)
- (2) large-mass ratios/high spin: amplitude problems
- (3) problems are extreme for high-spin EMRL waves
- (4) more flexible fit-template needed
- (5) improve/check over all datasets (SXS & BAM for large mass-ratios & consistency with EMRL)

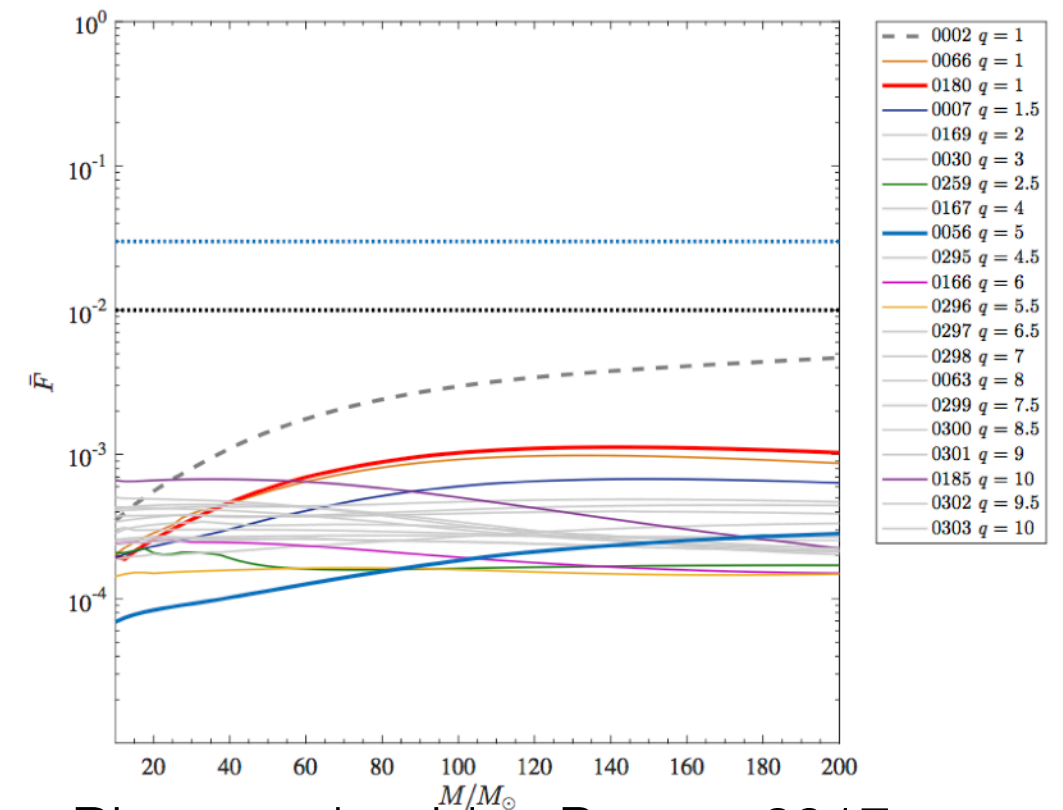
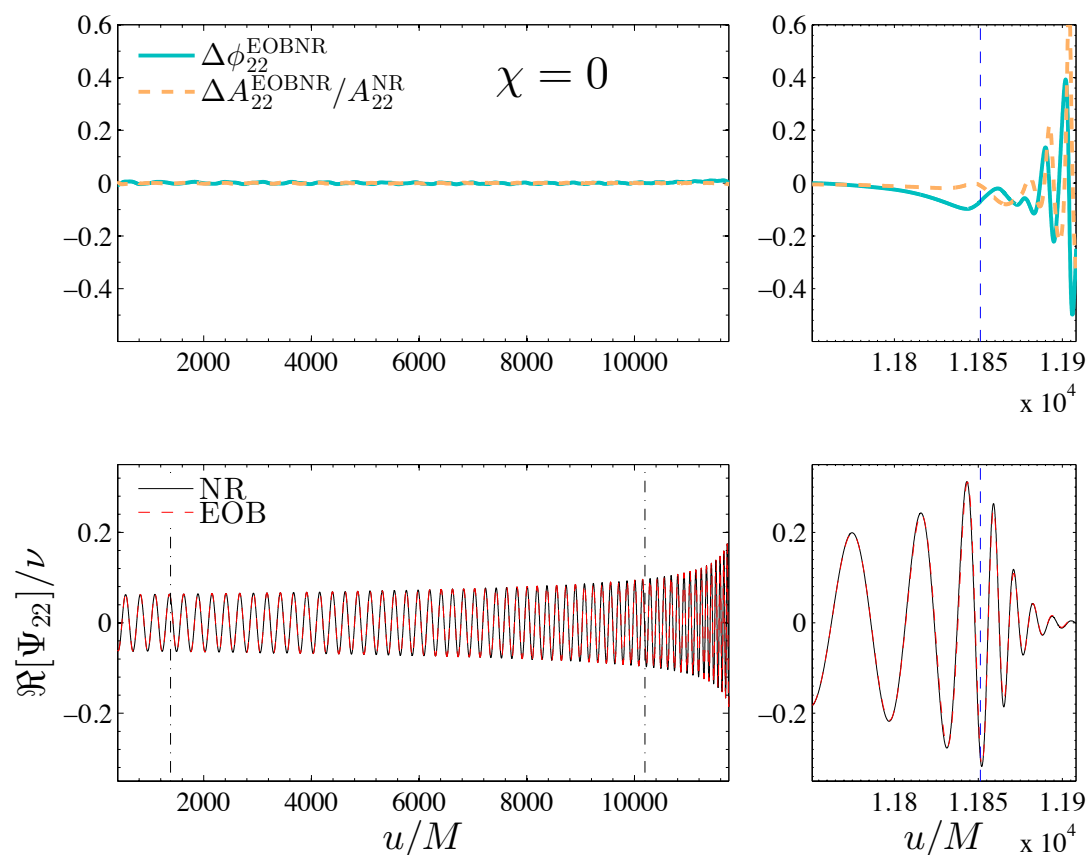
TEOBResumS point-mass potential



From EOB/NR-fitting:

$$a_6^c(\nu) = 3097.3\nu^2 - 1330.6\nu + 81.38$$

$$\bar{F}(M) \equiv 1 - F = 1 - \max_{t_0, \phi_0} \frac{\langle h_{22}^{\text{EOB}}, h_{22}^{\text{NR}} \rangle}{\|h_{22}^{\text{EOB}}\| \|h_{22}^{\text{NR}}\|},$$



Nagar, Riemenschneider, Pratten 2017

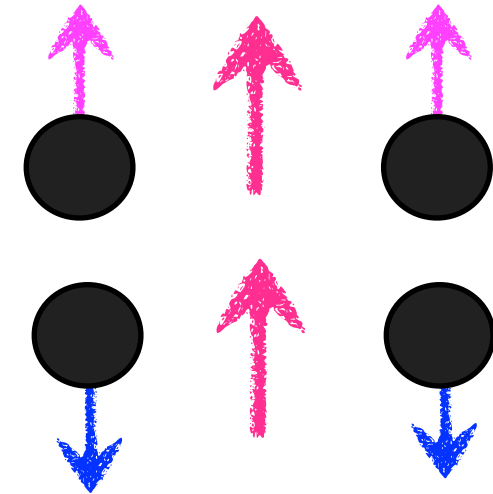
Spinning BBHs

Spin-orbit & spin-spin couplings

(i) Spins **aligned** with **L**: **repulsive** (slower) **L-o-n-g-e-r INSPIRAL**

(ii) Spins **anti-aligned** with **L**: **attractive** (faster) **shorter INSPIRAL**

(iii) Misaligned spins: precession of the orbital plane (**waveform modulation**)



$$\chi_{1,2} = \frac{c \mathbf{S}_{1,2}}{G m_{1,2}^2}$$

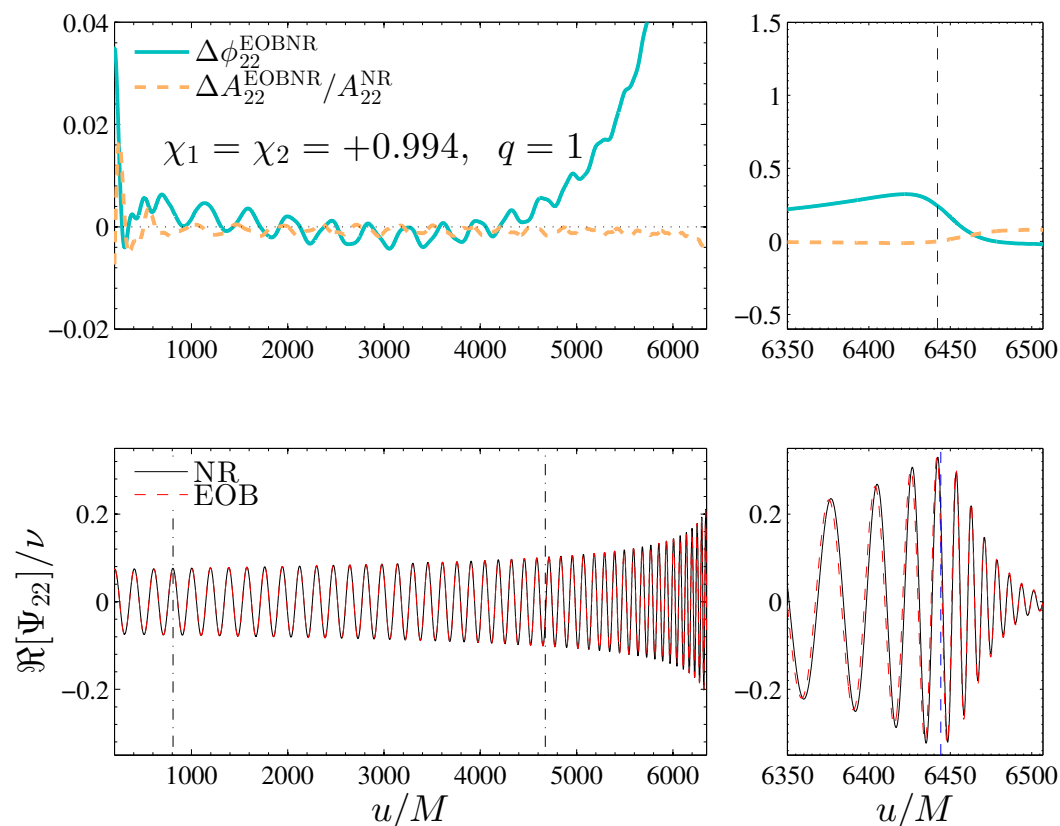
EOB/NR agreement: sophisticated (though rather simple) model for spin-aligned binaries

Damour&Nagar, PRD90 (2014), 024054 (Hamiltonian)

Damour&Nagar, PRD90 (2014), 044018 (Ringdown)

Nagar,Damour, Reisswig & Pollney, PRD 93 (2016), 044046

AEI model, SEOBNRv4, Bohe et al., arXiv:1611.03703v1 (PRD in press)



Spin-Spin in Kerr Hamiltonian

Particle: (μ, S_*)

Kerr black-hole: (M, S)

$$H_{\text{Kerr}} = H_{\text{orb}}^{\text{Kerr}} + H_{\text{SO}}^S(\mathbf{S}) + H_{\text{SO}}^{S_*}(\mathbf{S}_*)$$

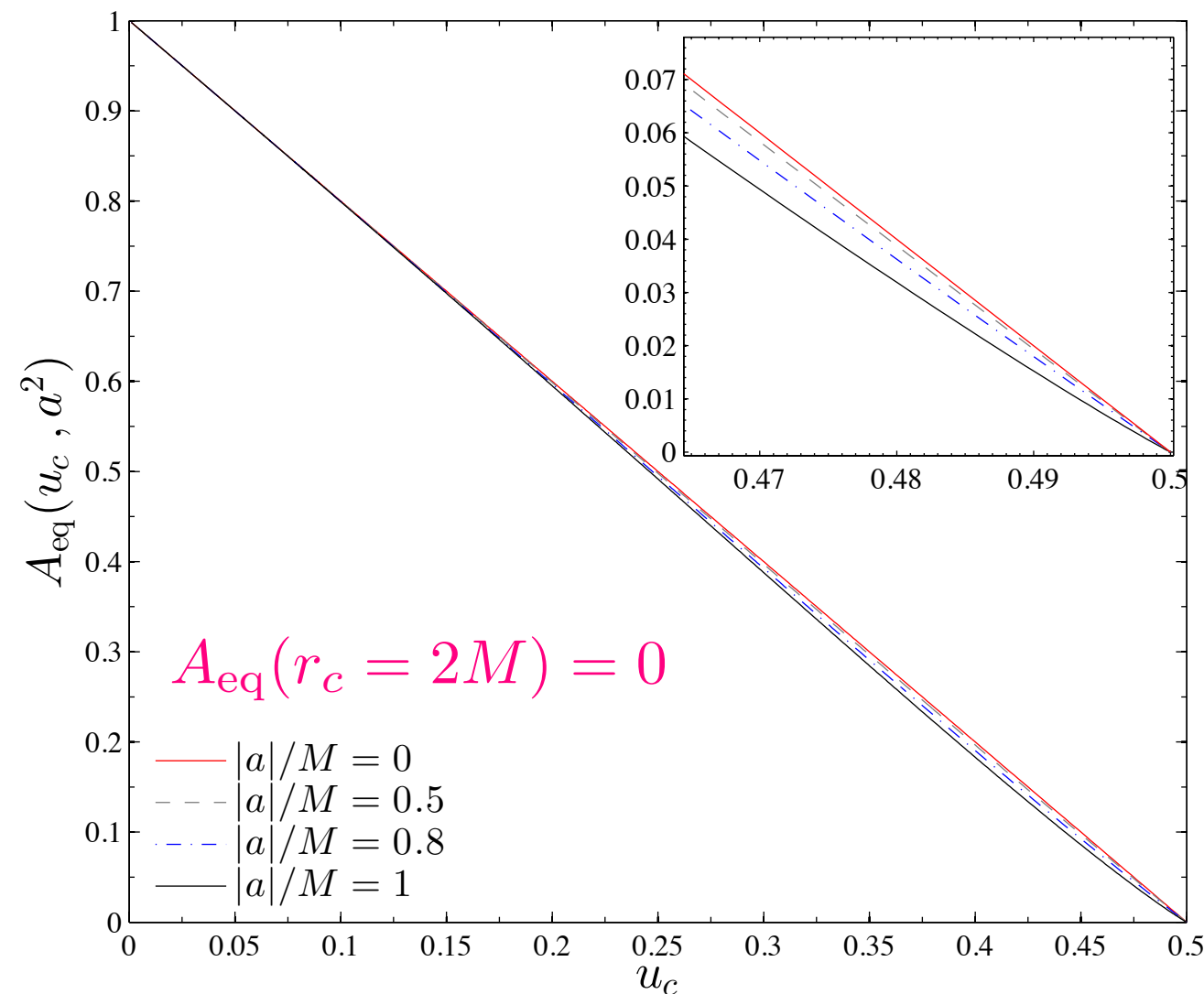
$$H_{\text{orb,eq}}^{\text{Kerr}}(r, p_r, p_\varphi) = \sqrt{A^{\text{eq}}(r) \left(\mu^2 + \frac{p_\varphi^2}{r_c^2} + \frac{p_r^2}{B^{\text{eq}}(r)} \right)}.$$

$$A_{\text{eq}}(r) \equiv \frac{\Delta(r)}{r_c^2} = \left(1 - \frac{2M}{r_c} \right) \frac{1 + \frac{2M}{r_c}}{1 + \frac{2M}{r}}$$

centrifugal radius

$$r_c^2 = r^2 + a^2 + \frac{2Ma^2}{r}$$

EOB: Identify a similar centrifugal radius in the comparable mass case and devise a similar deformation of A



Similar, though different, in SEOB

The effective Hamiltonian

$$\hat{H}_{\text{eff}} = \frac{g_S^{\text{eff}}}{r^3} \mathbf{L} \cdot \mathbf{S} + \frac{g_{S^*}^{\text{eff}}}{r^3} \mathbf{L} \cdot \mathbf{S}^* + \sqrt{A(1 + \gamma^{ij} p_i p_j + Q_4(p))}$$

with the structure

$$g_S^{\text{eff}} = 2 + \nu(\text{PN corrections}) + (\text{spin})^2 \text{corrections}$$

$$g_{S^*}^{\text{eff}} = \left(\frac{3}{2} + \text{test mass coupling} \right) + \nu(\text{PN corrections}) + (\text{spin})^2 \text{corrections}$$

$$A = 1 - \frac{2}{r} + \nu(\text{PN corrections}) + (\text{spin})^2 \text{corrections}$$

$$\gamma^{ij} = \gamma_{\text{Kerr}}^{ij} + \nu(\text{PN corrections}) + \dots$$

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = M^2 (X_1^2 \chi_1 + X_2^2 \chi_2)$$

$$X_i = m_i/M$$

$$\mathbf{S}^* = \frac{m_2}{m_1} \mathbf{S}_1 + \frac{m_1}{m_2} \mathbf{S}_2 = M^2 \nu (\chi_1 + \chi_2)$$

$$-1 \leq \chi_i \leq 1$$

THE TWO TYPES OF SPIN-ORBIT COUPLINGS

$$\hat{H}_{\text{SO}}^{\text{eff}} = G_S \mathbf{L} \cdot \mathbf{S} + G_{S^*} \mathbf{L} \cdot \mathbf{S}^* \quad G_S = \frac{1}{r^3} g_S^{\text{eff}}, \quad G_{S^*} = \frac{1}{r^3} g_{S^*}^{\text{eff}}$$

In the Kerr limit, only **S-type gyro-gravitomagnetic ratio** enters:

$$g_S^{\text{eff}} = 2 \frac{r^2}{r^2 + a^2 \left[(1 - \cos^2 \theta) \left(1 + \frac{2}{r} \right) + 2 \cos^2 \theta \right] + \frac{a^4}{r^2} \cos^2 \theta} = 2 + \mathcal{O}[(\text{spin})^2]$$

PN calculations yield (in some spin gauge)[DJS08, Hartung&Steinhoff11, Nagar11, Barausse&Buonanno11]

$$\begin{aligned} g_S^{\text{eff}} &= 2 + \frac{1}{c^2} \left\{ -\frac{1}{r} \frac{5}{8} \nu - \frac{33}{8} (\mathbf{n} \cdot \mathbf{p})^2 \right\} \\ &\quad + \frac{1}{c^4} \left\{ -\frac{1}{r^2} \left(\frac{51}{4} \nu + \frac{\nu^2}{8} \right) + \frac{1}{r} \left(-\frac{21}{2} \nu + \frac{23}{8} \nu^2 \right) (\mathbf{n} \cdot \mathbf{p})^2 + \frac{5}{8} \nu (1 + 7\nu) (\mathbf{n} \cdot \mathbf{p})^4 \right\}, \quad + \frac{1}{c^6} \frac{\nu c_3}{r^3} \\ g_{S^*}^{\text{eff}} &= \frac{3}{2} + \frac{1}{c^2} \left\{ -\frac{1}{r} \left(\frac{9}{8} + \frac{3}{4} \nu \right) - \left(\frac{9}{4} \nu + \frac{15}{8} \right) (\mathbf{n} \cdot \mathbf{p})^2 \right\} \\ &\quad + \frac{1}{c^4} \left\{ -\frac{1}{r^2} \left(\frac{27}{16} + \frac{39}{4} \nu + \frac{3}{16} \nu^2 \right) + \frac{1}{r} \left(\frac{69}{16} - \frac{9}{4} \nu + \frac{57}{16} \nu^2 \right) (\mathbf{n} \cdot \mathbf{p})^2 + \left(\frac{35}{16} + \frac{5}{2} \nu + \frac{45}{16} \nu^2 \right) (\mathbf{n} \cdot \mathbf{p})^4 \right\} + \frac{1}{c^6} \frac{\nu c_3}{r^3} \end{aligned}$$

This functions are resummed taking their Taylor-inverse

The NR-informed effective parameter makes the spin-orbit coupling stronger or weaker with respect to the straight analytical prediction

Spin-Spin within TEOBResumS

Define a "centrifugal radius": at LO reads

$$r_c^2 = r^2 + \hat{a}_0^2 \left(1 + \frac{2}{r} \right)$$

where:

$$\hat{a}_0^2 = \tilde{a}_1^2 + 2\tilde{a}_1\tilde{a}_2 + \tilde{a}_2^2$$

BBH case

or

$$\hat{a}_0^2 = C_{Q1}(\tilde{a}_1)^2 + 2\tilde{a}_1\tilde{a}_2 + C_{Q2}(\tilde{a}_2)^2$$

BNS case

$C_Q = 1$ is the BH case. In general, from I-Love-Q [Yagi-Yunes]

One verifies that once plugged in the EOB Hamiltonian and re-expanded one obtains the standard PN Hamiltonian @LO [e.g., cf. Levi-Steinhoff]

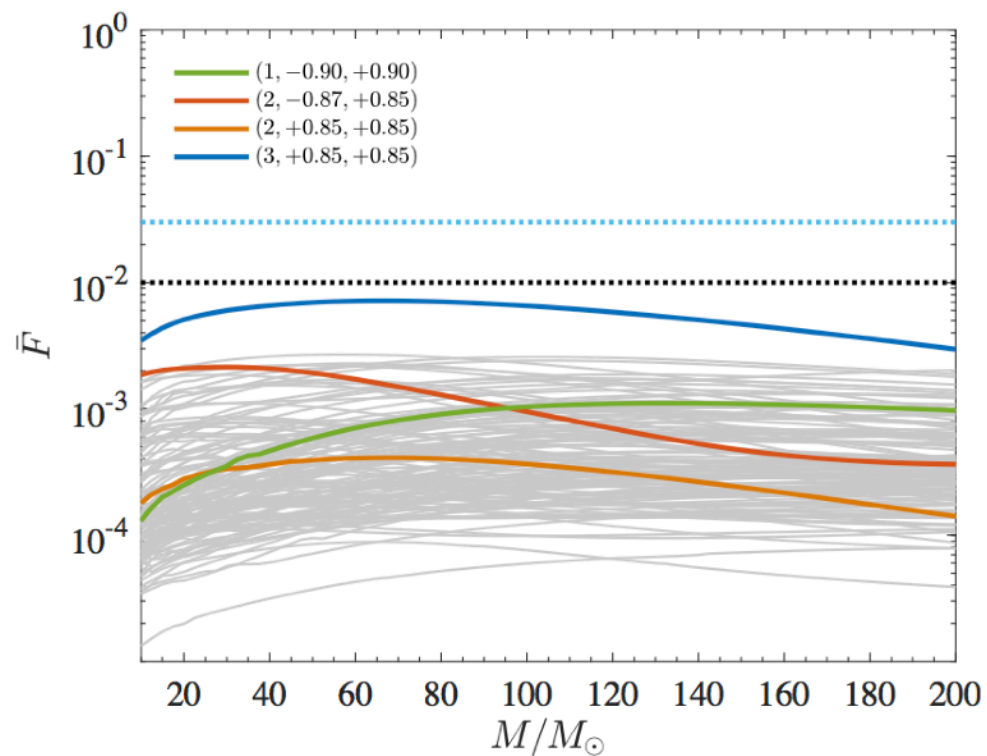
Similarly one can act on LO SS terms in the waveform & flux

$$r_c^2 = r^2 + \hat{a}_0^2 \left(1 + \frac{2}{r} \right) + \delta\hat{a}^2,$$

NLO contribution

$$A_{\text{eq}}(\nu, \chi_1, \chi_2) = A_{\text{orb}}^{\text{EOB}}(\nu, \kappa, r) \frac{1 + \frac{2}{r_c}}{1 + \frac{2}{r}}$$

TEOBResumS: spin-aligned + tides



- spin-orbit parameter informed by 30 BBH NR simulations
- **BEST faithfulness with all NR available (200 simulations)**
- Robust and simple
- Tides and spin-induced moment included (BNS)
- ONLY **publicly available** stand-alone EOB code

$$\bar{F}(M) \equiv 1 - F = 1 - \max_{t_0, \phi_0} \frac{\langle h_{22}^{\text{EOB}}, h_{22}^{\text{NR}} \rangle}{||h_{22}^{\text{EOB}}|| ||h_{22}^{\text{NR}}||},$$

Nagar, Bernuzzi, Del Pozzo et al., PRD98.104052

effective NNNLO spin-orbit “function”

$$c_3(\tilde{a}_A, \tilde{a}_B, \nu) = p_0 \frac{1 + n_1 \hat{a}_0 + n_2 \hat{a}_0^2}{1 + d_1 \hat{a}_0} + (p_1 \nu + p_2 \nu^2 + p_3 \nu^3) \hat{a}_0 \sqrt{1 - 4\nu} + p_4 (\tilde{a}_A - \tilde{a}_B) \nu^2, \quad (17)$$

$$\tilde{a}_{1,2} = X_{1,2} \chi_{1,2}$$

$$X_{1,2} \equiv \frac{m_{1,2}}{M}$$

$$\hat{a}_0 \equiv \frac{S + S_*}{M^2} = X_A \chi_A + X_B \chi_B = \tilde{a}_A + \tilde{a}_B$$

ONLY 2 EOBNR models
TEOBResumS
SEOBNRv4 (AEI)

See Rettegno, Martinetti, Nagar+2019,
arXiv:1911.10818

TEOBResumS + Post Adiabatic Approx

ODEs are slow: 1-2s for BNS waveforms (10Hz) not good for DA

Shared solution: ROMs (surrogate models. Fast but not flexible)

Are ROMs really needed?

EOB equations of motion

$$\frac{d\varphi}{dt} = \frac{1}{\nu \hat{H}_{\text{EOB}} \hat{H}_{\text{eff}}^{\text{orb}}} \left[A \frac{p_\varphi}{r_c^2} + \hat{H}_{\text{eff}}^{\text{orb}} \tilde{G} \right], \quad (1)$$

$$\begin{aligned} \frac{dr}{dt} = & \left(\frac{A}{B} \right)^{1/2} \frac{1}{\nu \hat{H}_{\text{EOB}} \hat{H}_{\text{eff}}^{\text{orb}}} \times \\ & \times \left[p_{r_*} \left(1 + 2z_3 \frac{A}{r_c^2} p_{r_*}^2 \right) + \hat{H}_{\text{eff}}^{\text{orb}} p_\varphi \frac{\partial \tilde{G}}{\partial p_{r_*}} \right], \end{aligned} \quad (2)$$

$$\frac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi, \quad (3)$$

$$\begin{aligned} \frac{dp_{r_*}}{dt} = & - \left(\frac{A}{B} \right)^{1/2} \frac{1}{2\nu \hat{H}_{\text{EOB}} \hat{H}_{\text{eff}}^{\text{orb}}} \left[A' + p_\varphi^2 \left(\frac{A}{r_c^2} \right)' + \right. \\ & \left. + z_3 p_{r_*}^4 \left(\frac{A}{r_c^2} \right)' + 2\hat{H}_{\text{eff}}^{\text{orb}} p_\varphi \tilde{G}' \right], \end{aligned} \quad (4)$$

$$\tilde{G} \equiv G_S S + G_{S_*} S_*$$

TEOBResumS + Post Adiabatic Approx

Post-adiabatic approximation (Damour & AN, 2007)

2PA: used to have eccentricity free ID for the EOB EoM

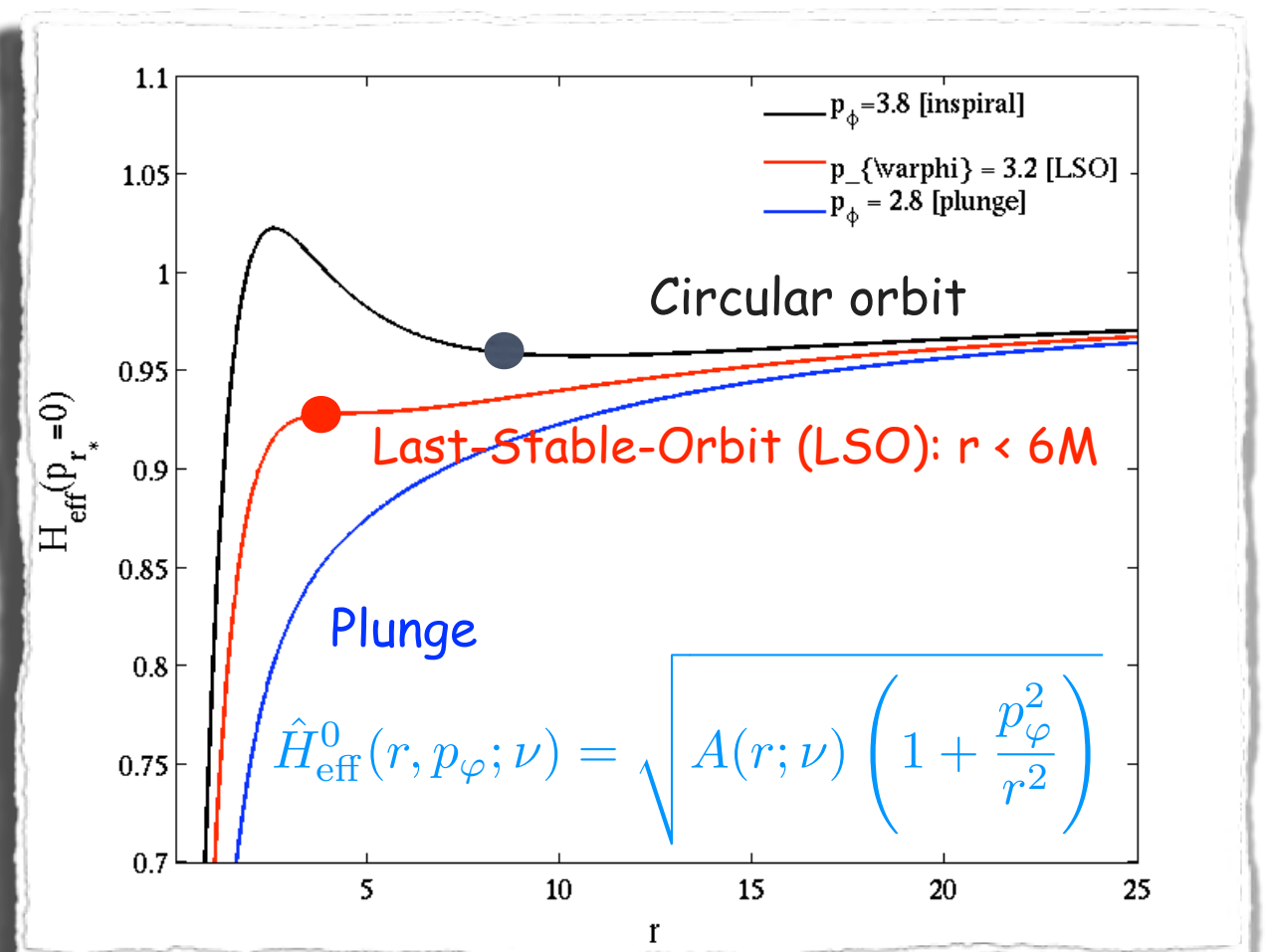
$$\hat{\mathcal{F}}_{\varphi}(r) = \sum_{n=0}^{\infty} \mathcal{F}_{2n+1}(r) \varepsilon^{2n+1}$$

$$p_{\varphi}^2(r) = j_0^2(r) \left(1 + \sum_{n=1}^{\infty} k_{2n}(r) \varepsilon^{2n} \right)$$

$$p_{r_*}(r) = \sum_{n=0}^{\infty} \pi_{2n+1}(r) \varepsilon^{2n+1}$$

Iterate up to nth order at a given radius to obtain the momenta with high accuracy

[Nagar&Rettegno, 2018]



TEOBResumS_rush

$$t = \int_{r_{\max}}^r dr (\hat{\partial}_{p_r} \hat{H})^{-1}$$

$$\varphi = \int_0^t dt \partial_{p_\varphi} \hat{H} = \int_{r_{\max}}^r dr \partial_{p_\varphi} \hat{H} (\partial_{p_r} \hat{H})^{-1}$$

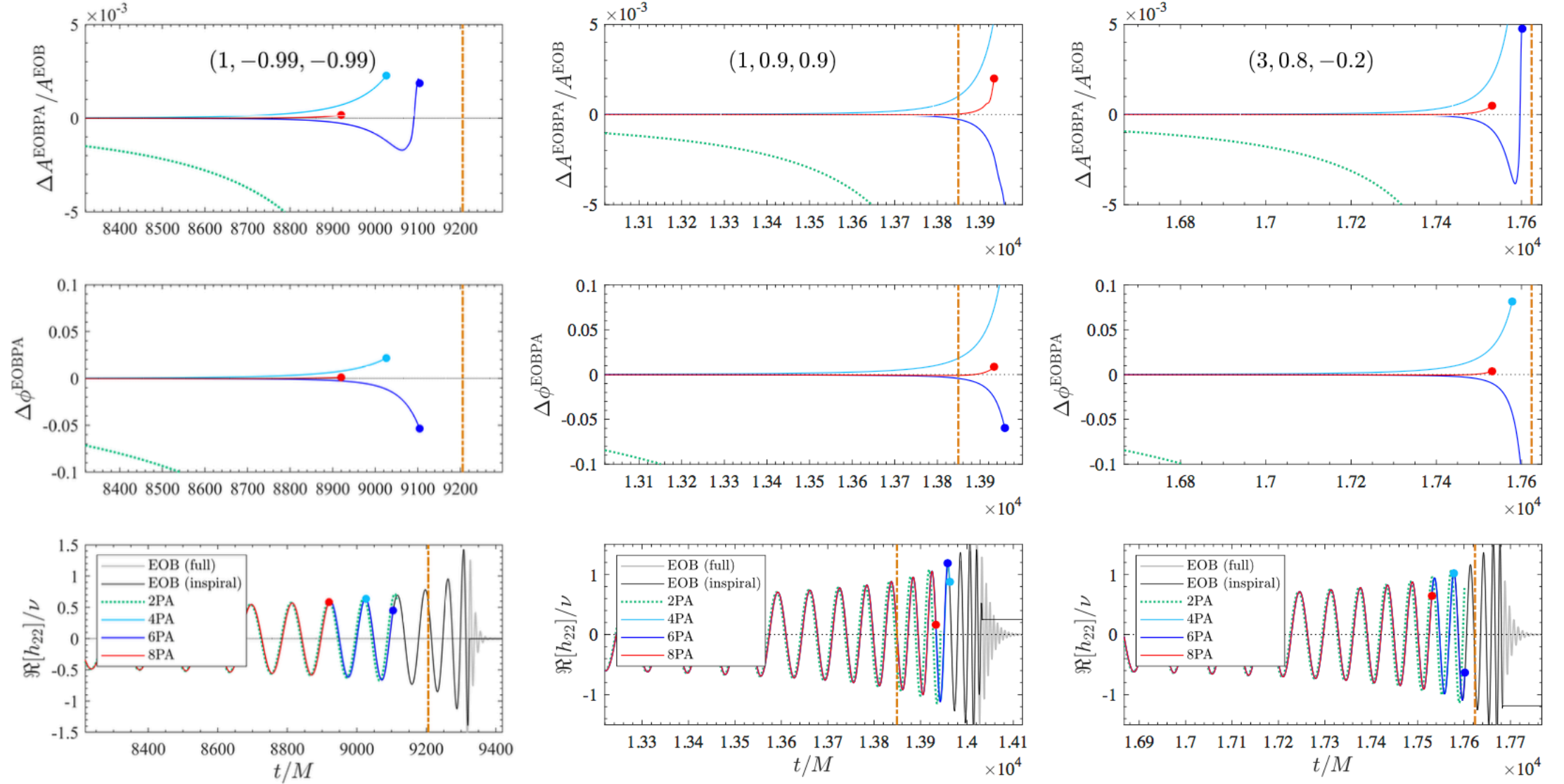


FIG. 1. Waveform comparison, $\ell = m = 2$ strain mode: EOB_{PA} inspiral (colors) versus EOB inspiral obtained solving the ODEs (black). The orange vertical line marks the EOB LSO crossing for $(1, -0.99, -0.99)$ and $(3, +0.80, -0.20)$, while it corresponds to $r = 6$ -crossing for $(1, +0.90, +0.90)$. The 4PA approximation already delivers an acceptable EOB/ EOB_{PA} agreement for both phase, ϕ , and amplitude, A . This is improved further by the successive approximations. At 8PA, the GW phase difference is $\lesssim 10^{-3}$ rad up to ~ 3 orbits before merger. The light-gray curve also incorporates the EOB-merger and ringdown.

TEOBResumS and GW150914

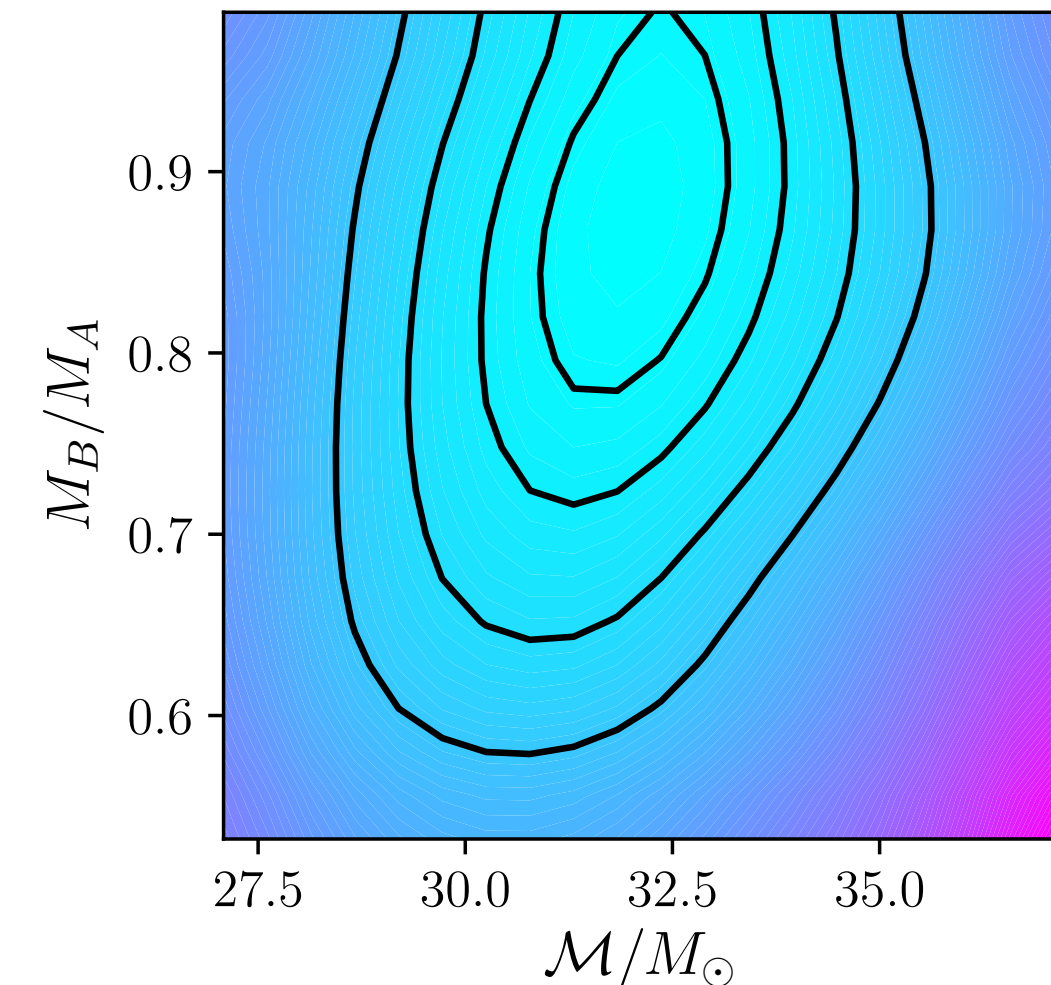
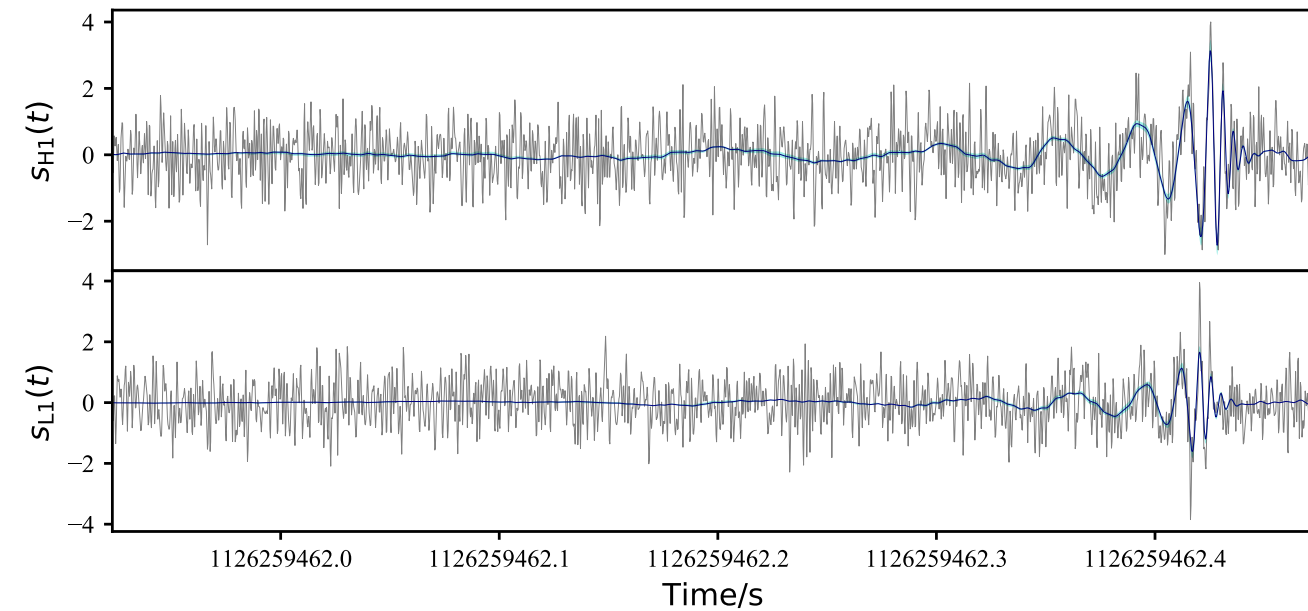
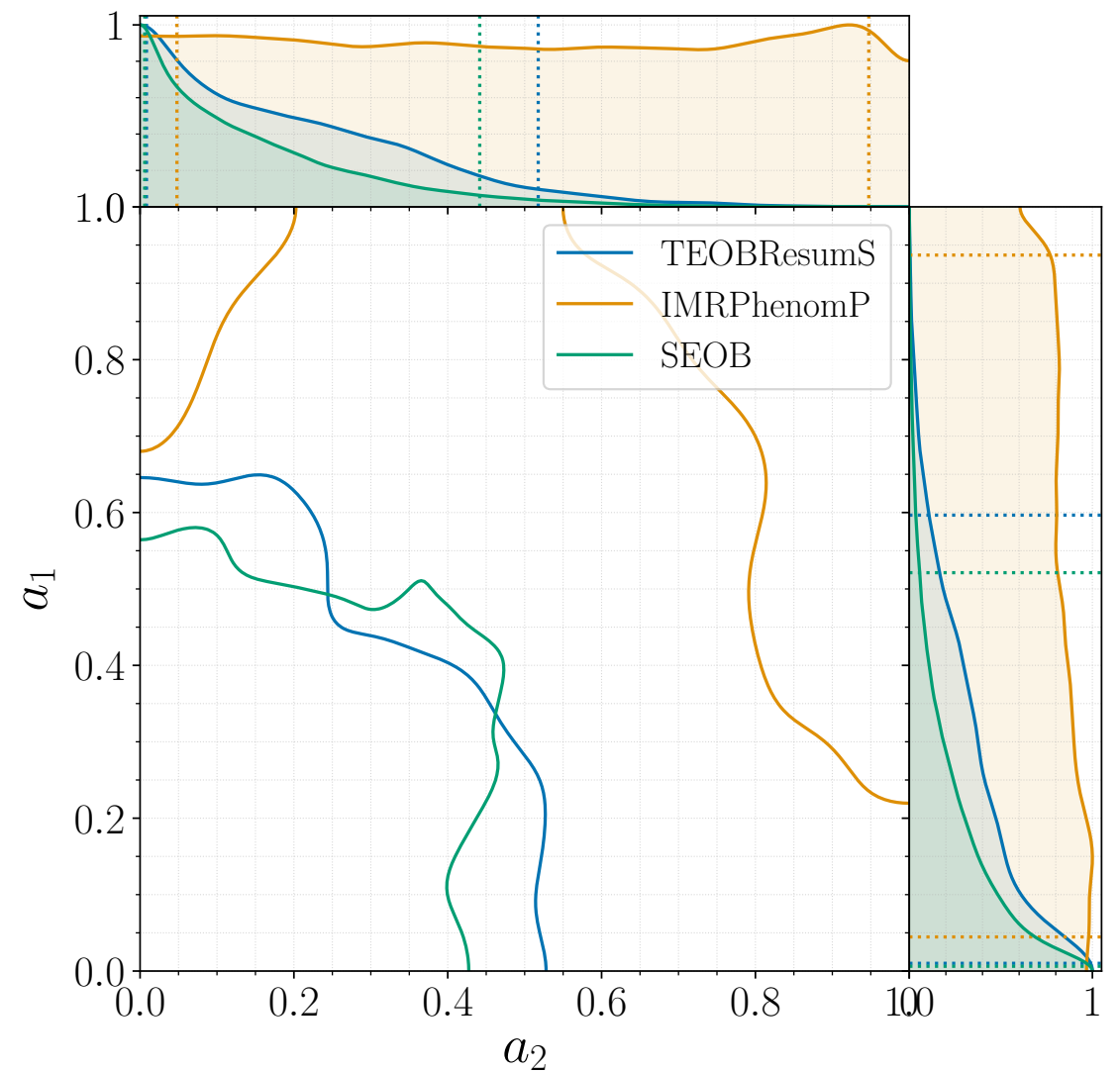
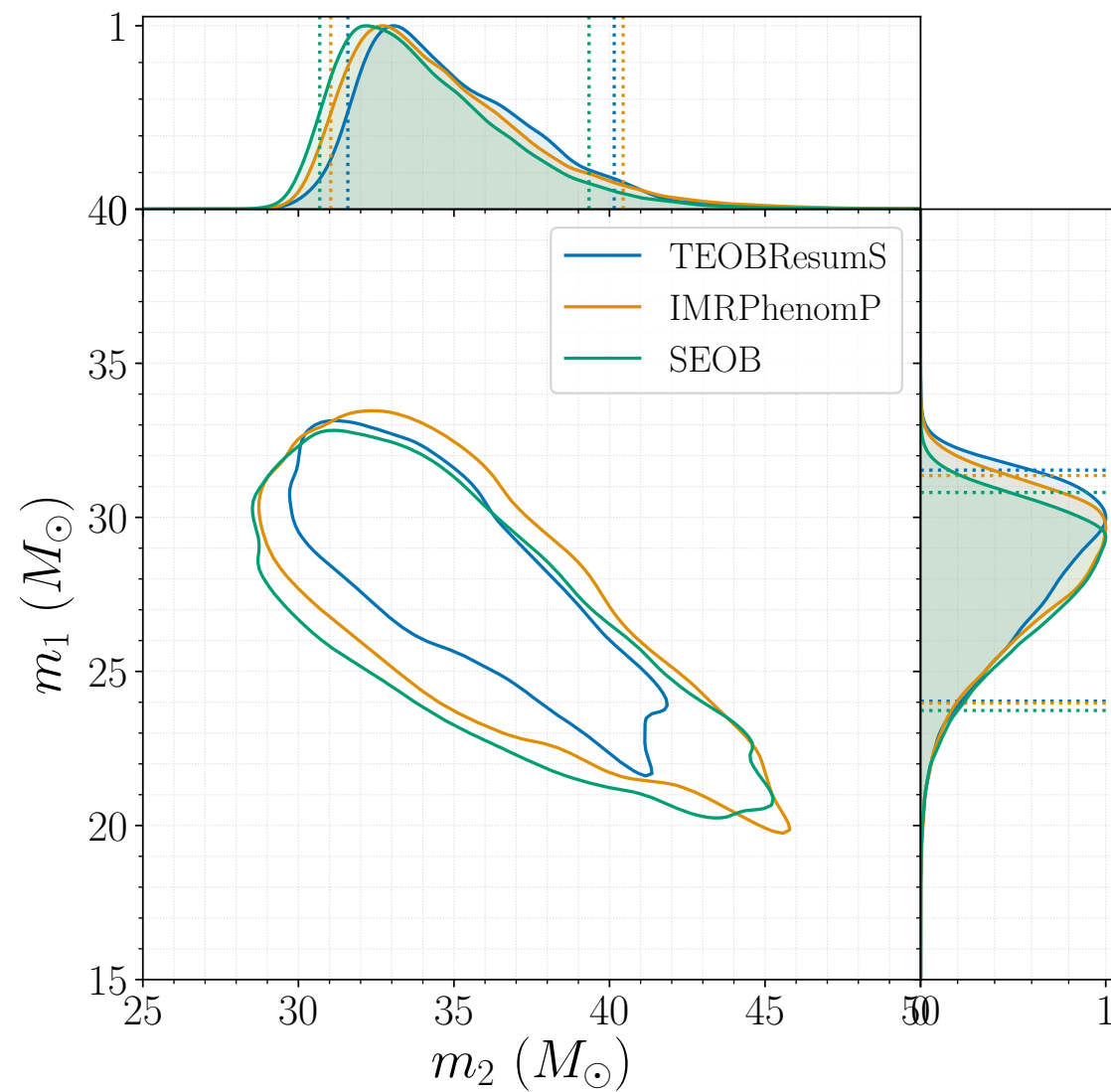


TABLE IV. Summary of the parameters that characterize GW150914 as found by `cpnest` and using `TEOBResumS` as template waveform, compared with the values found by the LVC collaboration [135]. We report the median value as well as the 90% credible interval. For the magnitude of the dimensionless spins $|\chi_A|$ and $|\chi_B|$ we also report the 90% upper bound. Note that we use the notation $\chi_{\text{eff}} \equiv \hat{a}_0$ for the effective spin, as introduced in Eq. (8).

	TEOBResumS	LVC
Detector-frame total mass M/M_\odot	$73.6^{+5.7}_{-5.2}$	$70.6^{+4.6}_{-4.5}$
Detector-frame chirp mass \mathcal{M}/M_\odot	$31.8^{+2.6}_{-2.4}$	$30.4^{+2.1}_{-1.9}$
Detector-frame remnant mass M_f/M_\odot	$70.0^{+5.0}_{-4.6}$	$67.4^{+4.1}_{-4.0}$
Magnitude of remnant spin \hat{a}_f	$0.71^{+0.05}_{-0.07}$	$0.67^{+0.05}_{-0.07}$
Detector-frame primary mass M_A/M_\odot	$40.2^{+5.1}_{-3.7}$	$38.9^{+5.6}_{-4.3}$
Detector-frame secondary mass M_B/M_\odot	$33.5^{+4.0}_{-5.5}$	$31.6^{+4.2}_{-4.7}$
Mass ratio M_B/M_A	$0.8^{+0.1}_{-0.2}$	$0.82^{+0.20}_{-0.17}$
Orbital component of primary spin χ_A	$0.2^{+0.6}_{-0.8}$	$0.32^{+0.49}_{-0.29}$
Orbital component of secondary spin χ_B	$0.0^{+0.9}_{-0.8}$	$0.44^{+0.50}_{-0.40}$
Effective aligned spin χ_{eff}	$0.1^{+0.1}_{-0.2}$	$-0.07^{+0.16}_{-0.17}$
Magnitude of primary spin $ \chi_A $	≤ 0.7	≤ 0.69
Magnitude of secondary spin $ \chi_B $	≤ 0.9	≤ 0.89
Luminosity distance d_L/Mpc	479^{+188}_{-235}	410^{+160}_{-180}

Nagar, Bernuzzi, Del Pozzo et al., PRD,
arXiv:1806.01772

TEOBResumS on GW150914



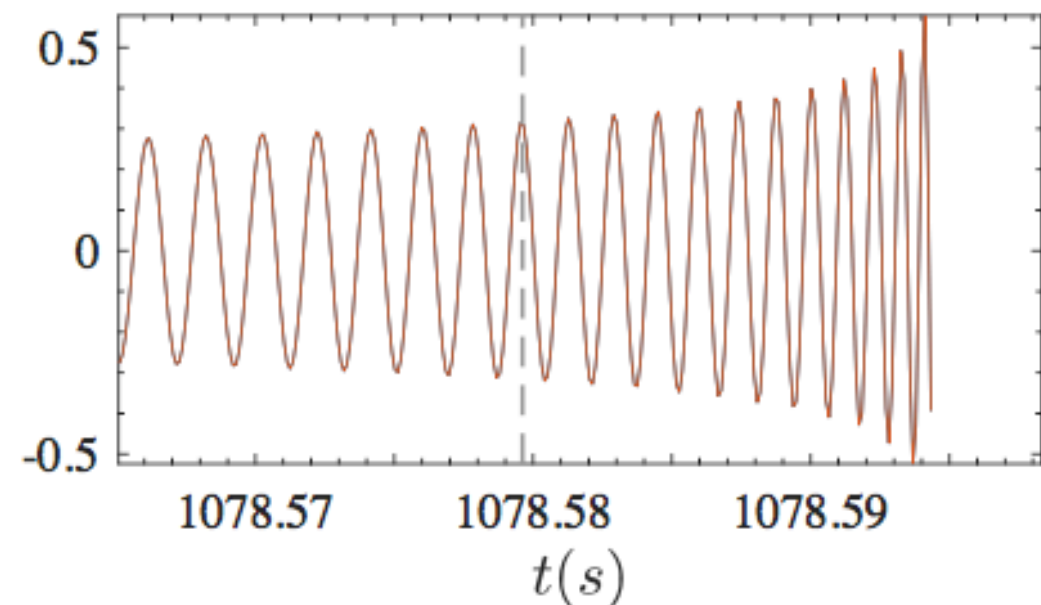
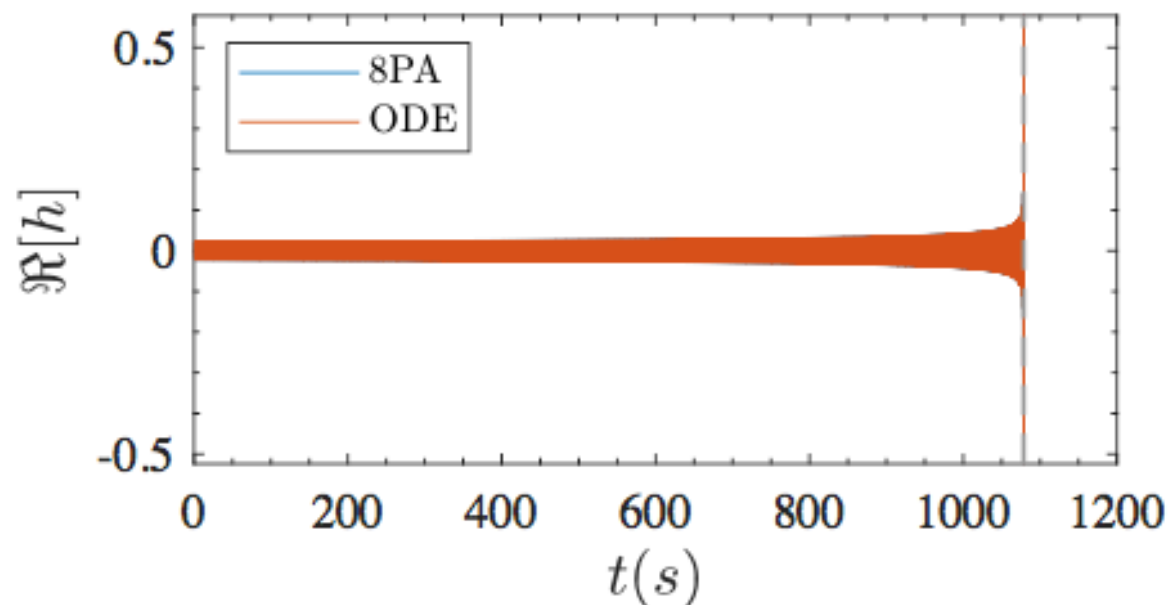
State of the art:

TEOBResumS: PA approx + ODE+LAL implementation

Neutron stars: tides & spin

TEOBResumS today [AN+, PRD98, 2018,104052]

- tidal effects + nonlinear-in-spin-effects (S^2, S^3, S^4, \dots) [AN+, PRD99, 2019,044007]
- analytically very complete model (almost final)
- $l=3$ GSF-informed + gravitomagnetic tides [Akcaay+, PRD, 2019, in press]
- checked with (state-of-the-art but short) NR simulations up to merger
- EFFICIENT due to the post-adiabatic approximation [AN & Rettegno PRD99, 2019 021501]
- no precession (yet!)

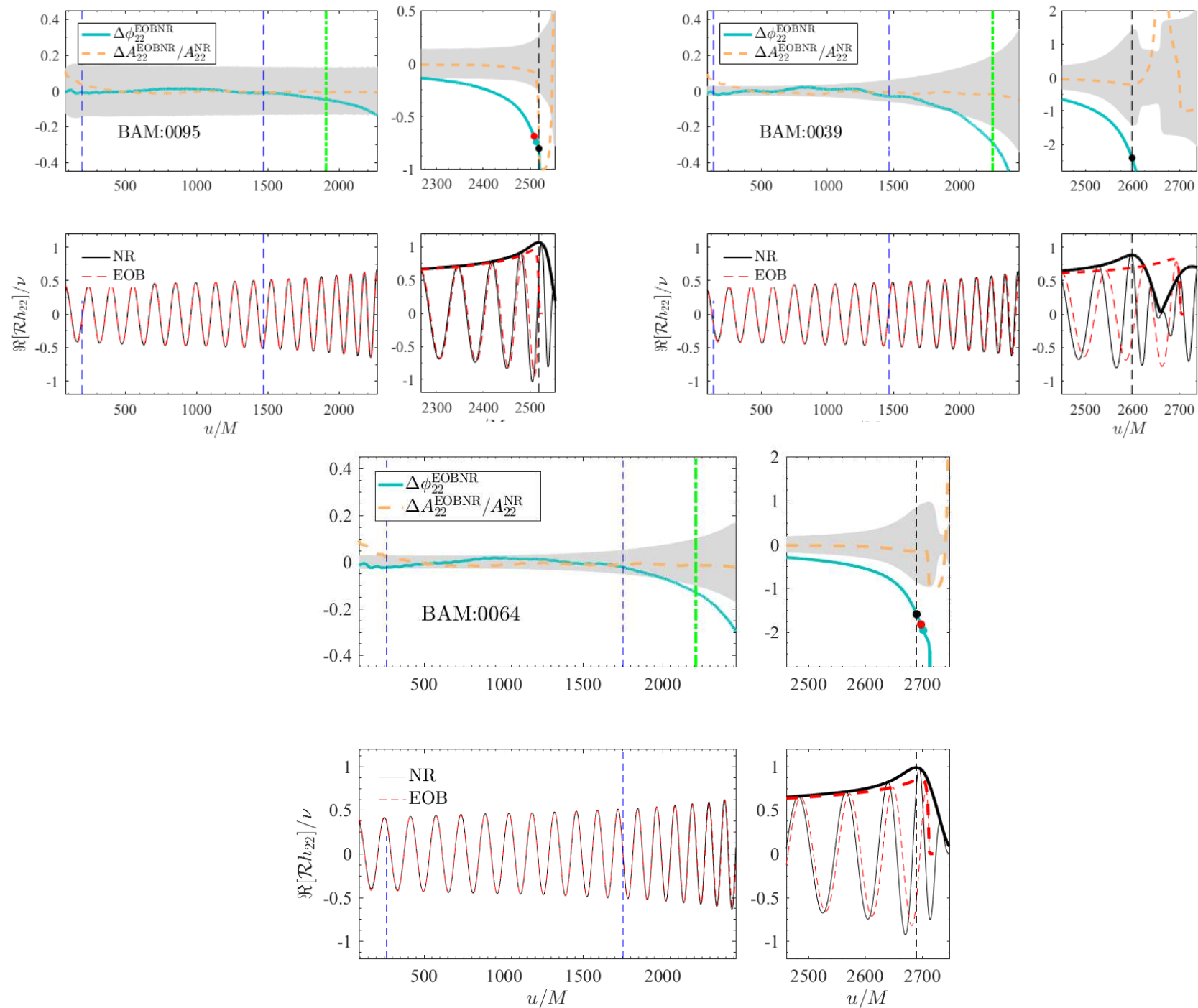


f_0 [Hz]	r_0	r_{\min}	N_r	Δr	τ_{8PA} [sec]	τ_{ODE} [sec]
20	112.81	12	500	0.20	0.03	0.53
10	179.02	12	830	0.20	0.05	1.1

No real need of EOB-surrogate!

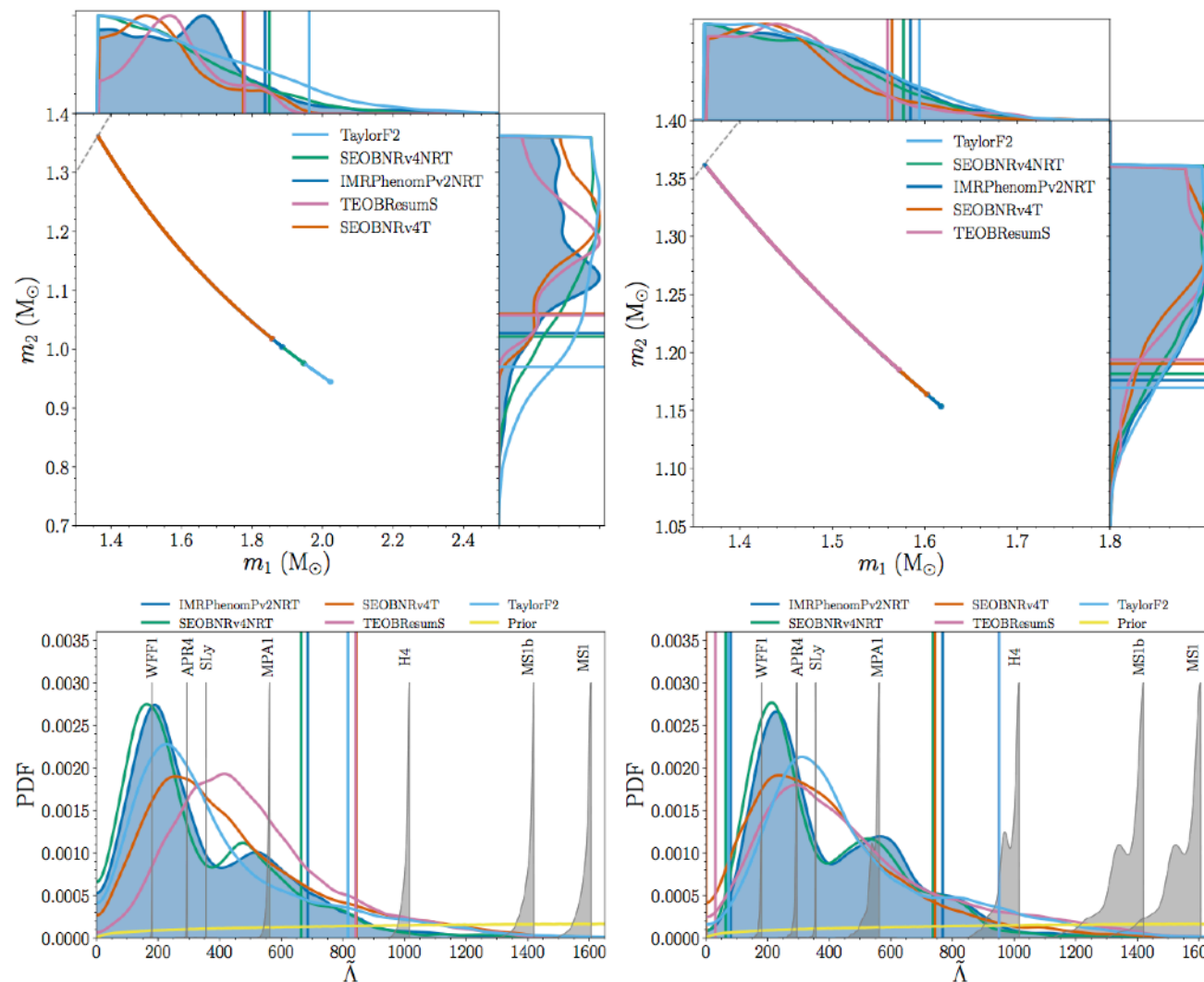
TEOBResumS vs NR: BNS

name	EOS	$M_{A,B}[M_\odot]$	$C_{A,B}$	$k_2^{A,B}$	κ_2^T	$\Lambda_2^{A,B}$	$\chi_{A,B}$	$C_{QA,QB}$
BAM:0095	SLy	1.35	0.17	0.093	73.51	392	0.0	5.491
BAM:0039	H4	1.37	0.149	0.114	191.34	1020.5	0.141	7.396
BAM:0064	MS1b	1.35	0.142	0.134	289.67	1545	0.0	8.396



GW170817- Parameter Estimation (LVC)

- Only existing EOB model independent from existing waveform models in LIGO/Virgo
- PE of the binary neutron star GW170817: arXiv:1811.12907 (GWTC-1)



Masses

Tidal polarizability
(EOS)

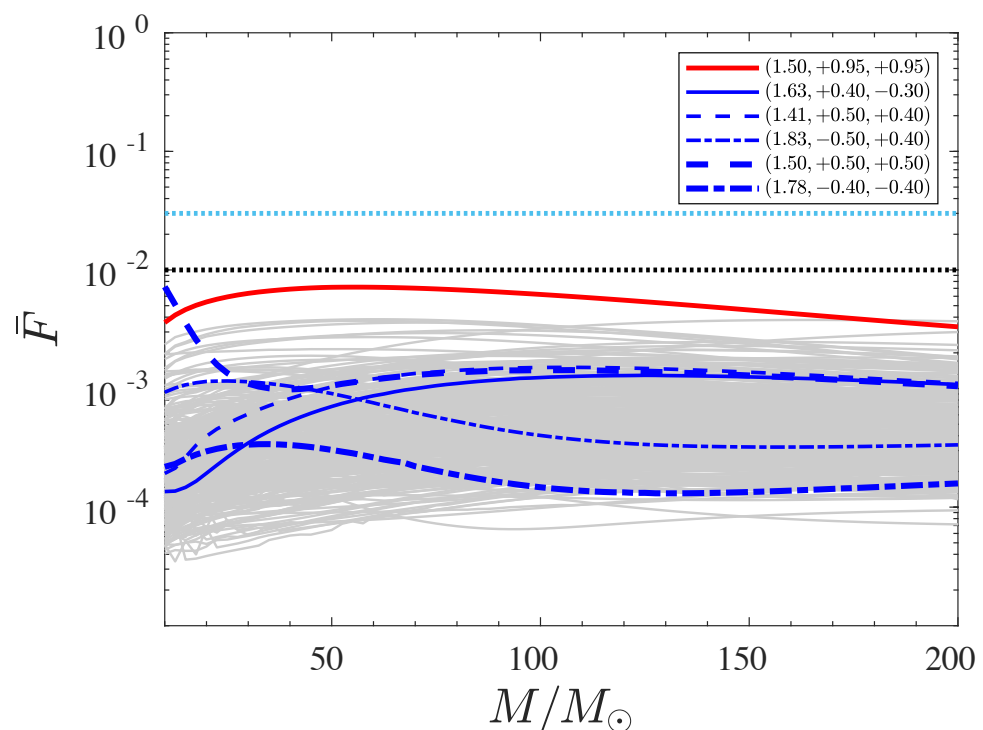
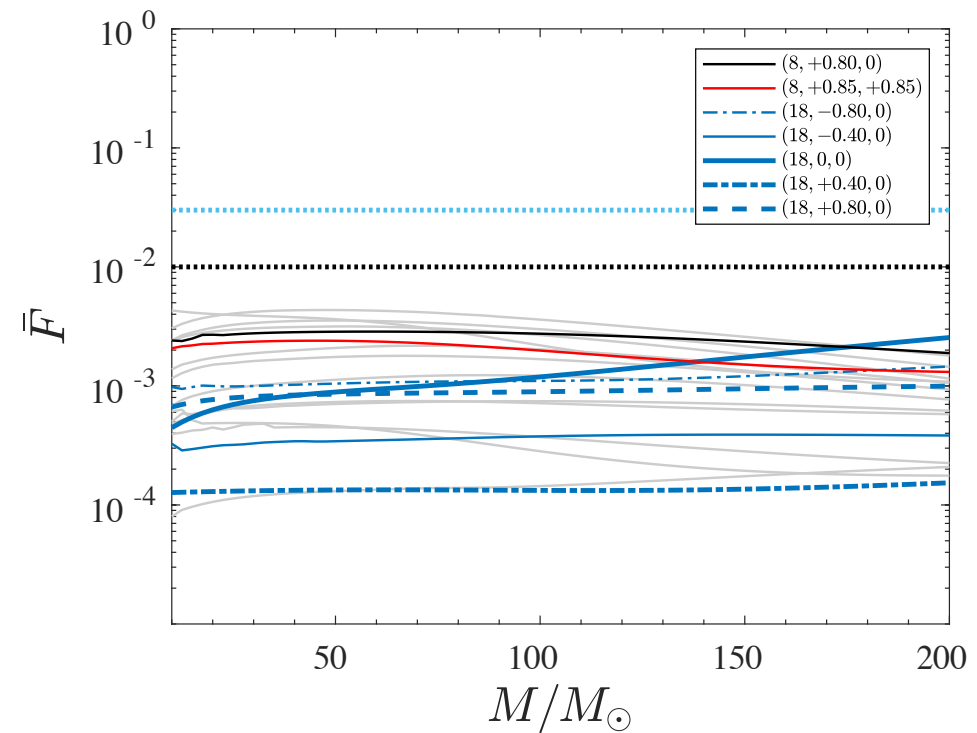
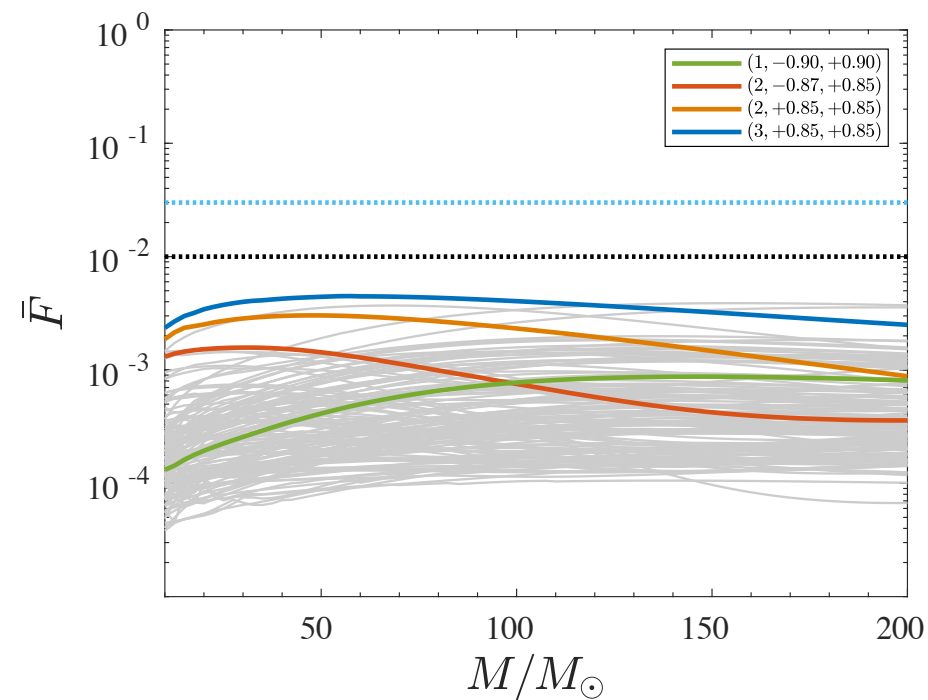
**BIASES ARE POSSIBLE USING
BAD TIDAL MODELS!!!!**

$$\tilde{\Lambda} = \frac{16}{13} \frac{(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{M^5}.$$

Recent development

Improved spin content in fluxes

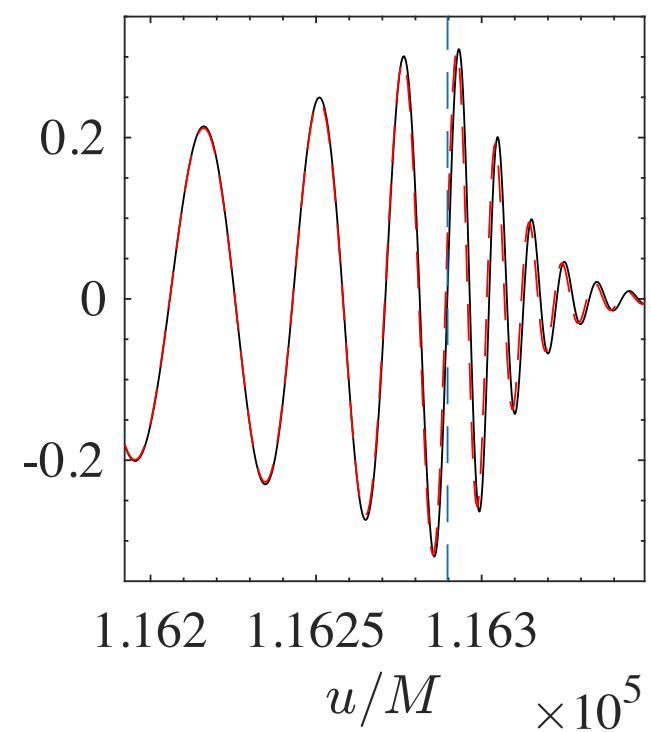
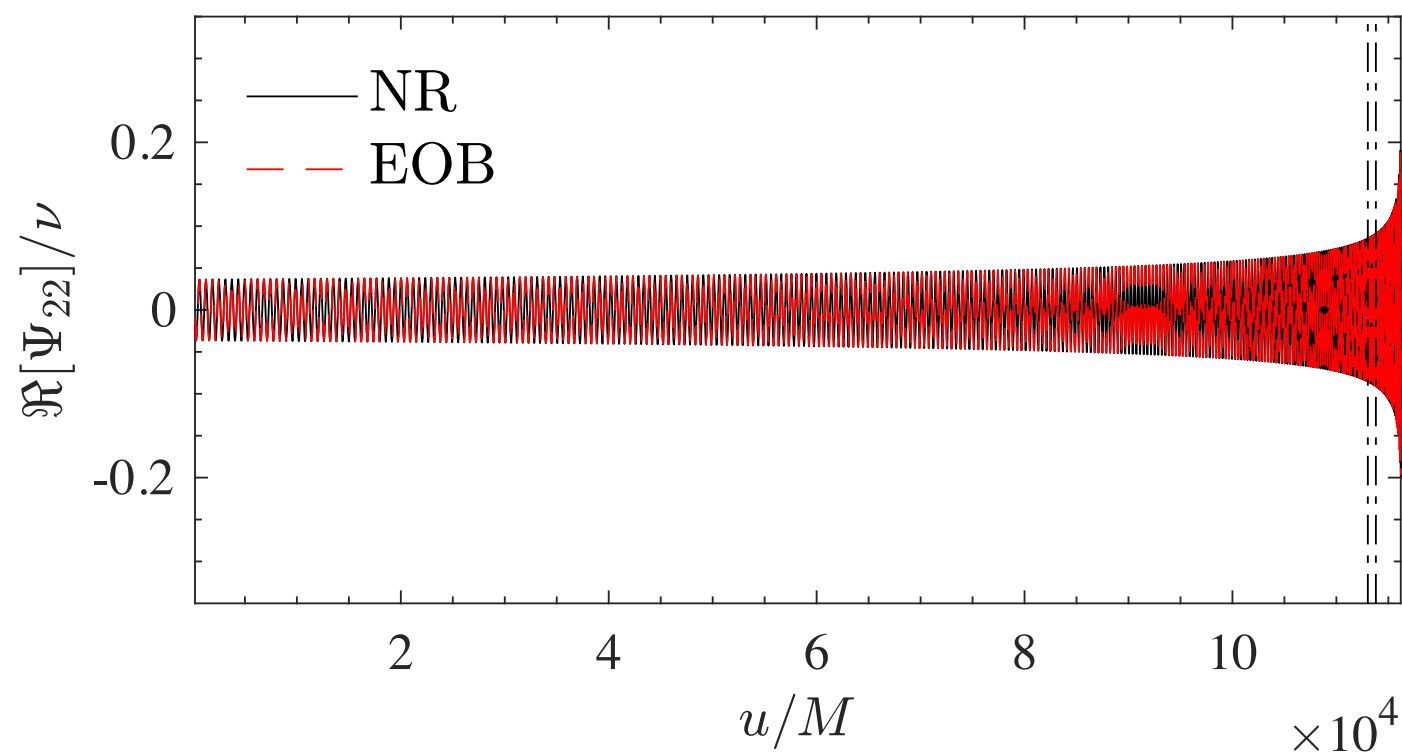
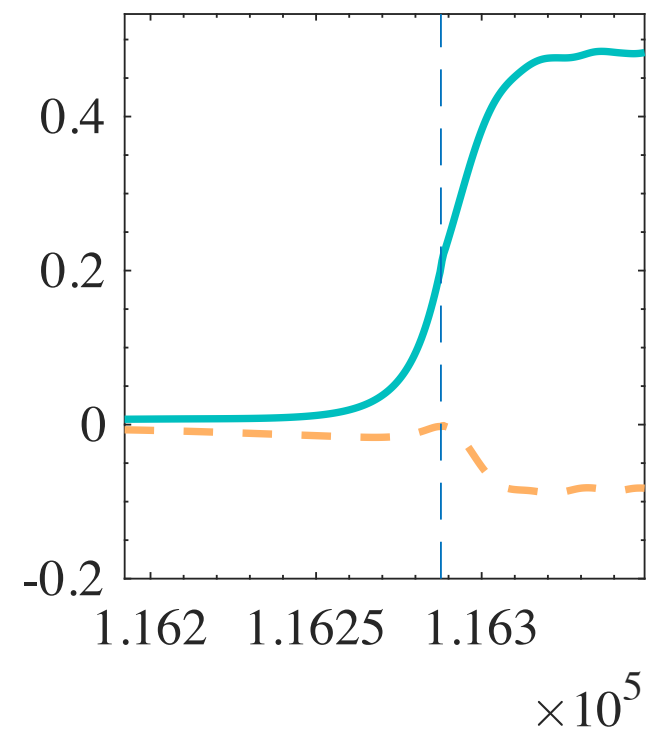
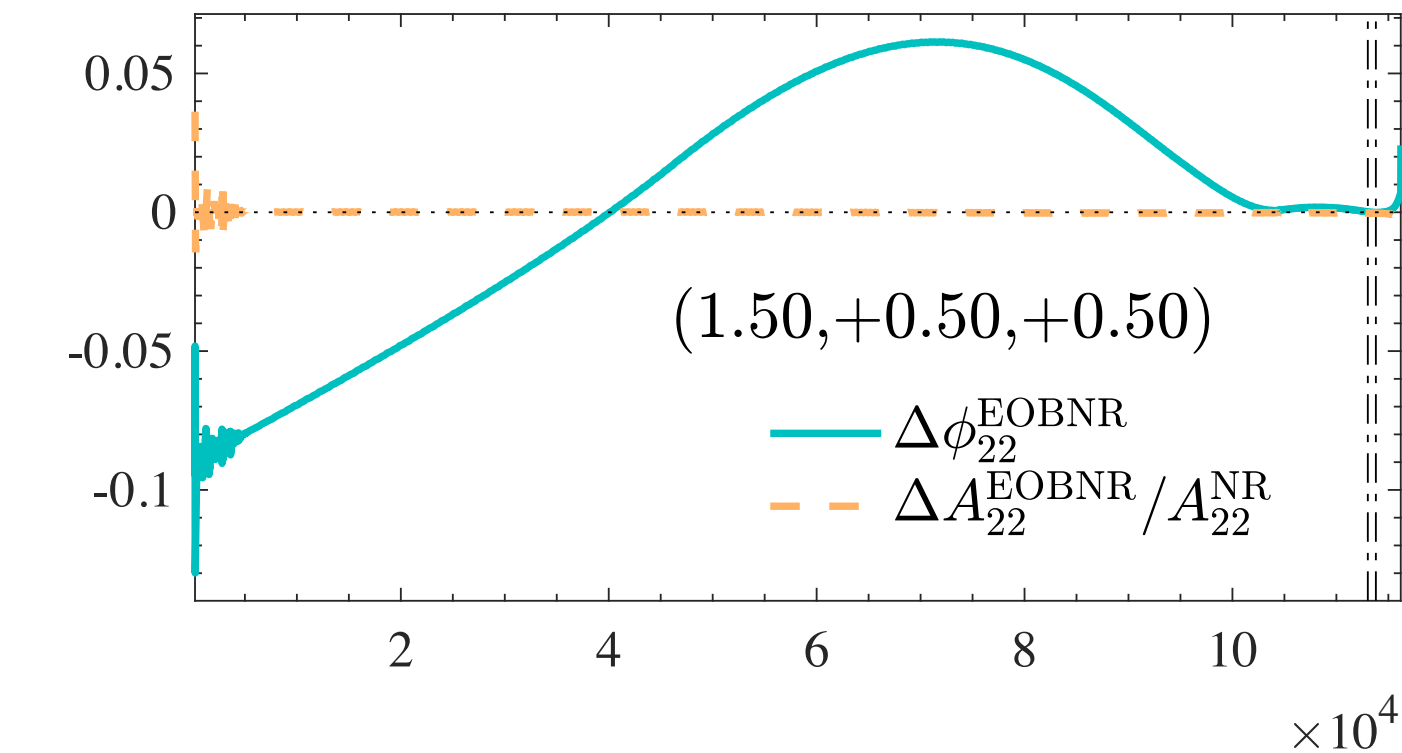
More robust resummation of waveform amplitudes



32 simulations to determine c_3

tested over 132+338 waveforms

Do we trust NR?



Conclusions

SEOB vs TEOB: consistent BUT different.

Analytic differences are being spelled out (MNR, in prep. 2019)

Spin sector very different!

TEOB is more efficient due to PA approx. Long inspirals.

No need of surrogate (e.g., is being used on BNS GW190426)

Good **analytic modeling** needed for reducing systematics. All current GW signal are going to be re-analyzed with TEOB

BBH+higher modes (no spin): arXiv:1904.09550

Higher modes with spin: in progress

Next challenge: eccentricity (in progress)