Coalescing compact binaries: the theory

Alessandro Nagar
INFN Torino and IHES

THEORY for CBC

- Interface between Analytical & Numerical Relativity for GW data-analysis
- 2-body problem in General Relativity

Analytical relativity (AR)
- Perturbation theory: PM, PN, EOB

Numerical Relativity (NR)
- (supercomputers)

GW data analysis

Challenges:
- physical completeness
- accuracy
- efficiency (AR vs NR)
- $10^7$ templates needed for a single event
Why waveform templates?

\[
\text{strain} = \frac{\delta L}{L}
\]

Symmetric mass ratio

\[
\nu = \frac{m_1 m_2}{(m_1 + m_2)^2} = 0.2466
\]

GW150914 parameters:

- \( m_1 = 35.7M_\odot \)
- \( m_2 = 29.1M_\odot \)
- \( M_f = 61.8M_\odot \)
- \( a_1 \equiv S_1/(m_1^2) = 0.31^{+0.48}_{-0.28} \)
- \( a_2 \equiv S_2/(m_2^2) = 0.46^{+0.48}_{-0.42} \)
- \( a_f \equiv \frac{J_f}{M_f^2} = 0.67 \)
- \( q \equiv \frac{m_1}{m_2} = 1.27 \)

Matched filtering: detection and parameter estimation

\[
\langle \text{output} | h_{\text{template}} \rangle = \int \frac{df}{S_n(f)} o(f) h^*_{\text{template}}(f)
\]

O2 events: GWTC-1: arXiv:1811.12907

GW150914: \( m_1 = 35.7M_\odot \), \( m_2 = 29.1M_\odot \), \( M_f = 61.8M_\odot \), \( a_1 \equiv S_1/(m_1^2) = 0.31^{+0.48}_{-0.28} \), \( a_2 \equiv S_2/(m_2^2) = 0.46^{+0.48}_{-0.42} \), \( a_f \equiv \frac{J_f}{M_f^2} = 0.67 \), \( q \equiv \frac{m_1}{m_2} = 1.27 \)

Matched filtering: detection and parameter estimation

\[
\langle \text{output} | h_{\text{template}} \rangle = \int \frac{df}{S_n(f)} o(f) h^*_{\text{template}}(f)
\]

Analytical formalism:
theoretical understanding
of the coalescence process
2-body problem in GR

Hamiltonian: conservative part of the dynamics

Radiation reaction: mechanical energy/angular momentum goes away in GWs and backreacts on the system.

The (closed) orbit CIRCULARIZES and SHRinks with time

Waveform

Perturbation Theory
Post-Newtonian (PN), Post-Minkowskian (PM)

Resummed
PN theory: EOB (LAPTOP)

Numerical Relativity: (SUPERCOMPUTERS)

Strong-field information
EOB, NR models

Complementary route: IMRPhenom models
PN_glue_NR, EOB_glue_NR hybrids (glued waveforms) to build phenomenological templates [Khan et al., 2015]

NR surrogate
EFFECTIVE-ONE-BODY (EOB) approach to the general relativistic two-body problem

(Buonanno-Damour 99, 00, Damour-Jaranowski-Schäfer 00, Damour 01, Damour-Nagar 07, Damour-Iyer-Nagar 08)

key ideas:

1. Replace two-body dynamics \((m_1, m_2)\) by dynamics of a particle \((\mu \equiv m_1 m_2 / (m_1 + m_2))\) in an effective metric \(g_{\mu \nu}^{\text{eff}}(u)\), with

\[
u \equiv \frac{GM}{c^2 R},\quad M \equiv m_1 + m_2\]

2. Systematically use RESUMMATION of PN expressions (both \(g_{\mu \nu}^{\text{eff}}\) and \(F_{RR}\)) based on various physical requirements

3. Require continuous deformation w.r.t.

\[
u \equiv \frac{\mu}{M} \equiv \frac{m_1 m_2}{(m_1 + m_2)^2}\]

in the interval \(0 \leq \nu \leq \frac{1}{4}\)
**STRUCTURE OF THE EOB FORMALISM**

**EOB Hamiltonian**
\[ H_{\text{EOB}} \]

**EOB Rad. Reac. force**
\[ \hat{f}_\phi \]

**EOB waveform**

\[ h_{\ell m} = \theta(t_m - t) h_{\ell m}^{\text{insplunge}}(t) + \theta(t - t_m) h_{\ell m}^{\text{ringdown}}(t) \]

+ **GSF** + EOB based on Post-Minkowskian approximation

**Matching at merger time**
\[ h_{\ell m}^{\text{ringdown}}(t) = \sum_N C_N^- e^{-\sigma_N^+(t-t_m)} \]

**PN dynamics**
(DD81,D82,DJS01,IF03,BDIF04)

Resummed (BD99)

**PN rad losses**
WW76, BDIWW95, BDEFI05

Resummed (DIS98)

**PN waveform**
BD89, B95&05, ABIQ04,

Resummed (DN07,DIN08)

**Resummed (BD99)**

**Resummed (DIS98)**

**Resummed (DN07,DIN08)**

**BH perturbations**
RW57, Z70, Z72

**QNMs spectrum**
\[ \sigma_N = \alpha_N + i\omega_N \]

**EOB Rad. Reac. force**
\[ \hat{f}_{\phi} \]

**Factorized waveform**

\[ h_{\ell m} = h_{\ell m}^{(N,e)} \hat{h}_{\ell m}^{(e)} \]

\[ \hat{h}_{\ell m}^{(e)} = \hat{S}_{\ell m}^{(e)} T_{\ell m} e^{i\delta_{\ell m}} \rho_{\ell m} \]

**Matching at merger time**
\[ h_{\ell m}^{\text{ringdown}}(t) = \sum_N C_N^- e^{-\sigma_N^+(t-t_m)} \]

**BNS: tides**
(Love numbers)

\[ \Omega = \frac{d\Phi}{dt} = \frac{\partial H_{\text{EOB}}}{\partial p_{\phi}}, \]

\[ \Omega = \frac{dr}{dt} = \left( \frac{A}{B} \right)^{1/2} \frac{\partial H_{\text{EOB}}}{\partial r}, \]

\[ \Omega = \frac{dp_{\phi}}{dt} = \hat{f}_{\phi}. \]

**Phenomenological fit**
EOB based on Post-Minkowskian approximation
Extreme-mass-ratio limit (2007)

- Laboratory to learn each physical element entering the coalescence
- Accurate waveform computation using Regge-Wheeler-Zerilli (Schwarzschild) or Teukolsky (Kerr) perturbation equations

- Several aspects of the phenomenon explored in detail
- Several papers with Bernuzzi+ (multipoles, GW-recoil, spin etc.): Teukode

Gravitational recoil in nonspinning black-hole binaries: The span of test-mass results

Alessandro Nagar

RWZ-extrapolated final recoil velocity and “full” NR calculations
EOBNR Models

**AEI (Ligo): LAL**
- SEOBNRv4 (spin-aligned)
- SEOBNRv4_HM (spin-aligned, 22,21,33,44,55 modes)
- SEOBNRv4P_HM (precessing spins, 22,21,33,44,55 modes)
- SEOBNRv4T (tides)

**(Virgo): standalone C & LAL implementation**
- TEOBResumS (spin-aligned, tides, BBH, BNS, BHNS)
- TEOBiResumS (higher modes, in progress)

Differences:
- Hamiltonian (gauge + spin sector. Spin-spin)
- Resummation of the interaction potential
- Radiation reaction
- Effective representation of merger and post merger
- ESSENTIALLY: different deformation of the test-mass limit
EOB Hamiltonian

\[ H_{\text{EOB}} = M \sqrt{1 + 2\nu \left( \hat{H}_{\text{eff}} - 1 \right)} \]

All functions are a \textit{-dependent deformation} of the Schwarzschild ones

\[ A(r) = 1 - 2u + 2\nu u^3 + a_4 \nu u^4 \]

\[ A(r)B(r) = 1 - 6\nu u^2 + 2(3\nu - 26)\nu u^3 \]

\[ u = \frac{GM}{c^2 R} \]

\[ a_4 = \frac{94}{3} - \frac{41}{32}\pi^2 \simeq 18.6879027 \]

Simple effective Hamiltonian:

\[ \hat{H}_{\text{eff}} \equiv \sqrt{p_{r*}^2 + A(r) \left( 1 + \frac{p_\varphi^2}{r^2} + z_3 \frac{p_{r*}^4}{r^2} \right)} \]

\[ p_{r*} = \left( \frac{A}{B} \right)^{1/2} p_r \]

\[ \text{Crucial EOB radial potential} \]

\[ \text{Contribution at 3PN} \]
4PN analytically complete + 5PN logarithmic term in the $A(u)$ function:

$$A_{5\text{PN}}^{\text{Taylor}} = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\nu u^4 + \nu[a^c_5(\nu) + a^c_5 \ln \ln u]u^5 + \nu[a^c_6(\nu) + a^c_6 \ln \ln u]u^6$$

\[
\begin{align*}
    a^\log_5 &= \frac{64}{5} \\
    a^c_5 &= a^c_{50} + \nu a^c_{51} \\
    a^c_{50} &= -\frac{4237}{60} + \frac{2275}{512}\pi^2 + \frac{256}{5}\log(2) + \frac{128}{5}\gamma \\
    a^c_{51} &= -\frac{221}{6} + \frac{41}{32}\pi^2 \\
    a^\log_6 &= -\frac{7004}{105} - \frac{144}{5}\nu
\end{align*}
\]

5PN logarithmic term (analytically known)

4PN fully known ANALYTICALLY!

State-of-the-art EOB potential (5PN-resummed):

$$A(u; \nu, a^c_6) = P_5^1[A_{5\text{PN}}^{\text{Taylor}}(u; \nu, a^c_6)]$$
Resummation of the waveform (and flux) multipole by multipole (CRUCIAL!)

[Damour&Nagar 2007, Damour, Iyer, Nagar 2008]

The "Tail factor"
Effective source:
EOB (effective) energy (even-parity modes)
EOB angular momentum (odd-parity modes)

Next-to-quasi-circular correction
Newtonian × PN × NQC

Remnant phase and modulus corrections:
"improved" PN series

Resums an infinite number of leading logarithms
in tail effects (hereditary contributions)
Factorize the fundamental QNM, fit what remains
\[ h(\tau) = e^{\sigma_1 \tau - i\phi_0} \bar{h}(\tau) \]
\[ \bar{h}(\tau) \equiv A_{\bar{h}} e^{i\phi_{\bar{h}}(\tau)} \]

\[ A_{\bar{h}}(\tau) = c_1^A \tanh(c_2^A \tau + c_3^A) + c_4^A, \]
\[ \phi_{\bar{h}}(\tau) = -c_1^\phi \ln \left( \frac{1 + c_3^\phi e^{-c_2^\phi \tau} + c_4^\phi e^{-2c_2^\phi \tau}}{1 + c_3^\phi + c_4^\phi} \right) \]

\[ c_2^A = \frac{1}{2} \alpha_{21}, \quad \alpha_{21} = \alpha_2 - \alpha_1 \]
\[ c_4^A = \tilde{A}_{22}^{\text{mrg}} - c_1^A \tanh(c_3^A), \]
\[ c_1^A = \tilde{A}_{22}^{\text{mrg}} \alpha_1 \frac{\cosh^2(c_3^A)}{c_2^A}, \]
\[ c_1^\phi = \Delta \omega \frac{1 + c_3^\phi + c_4^\phi}{c_2^\phi (c_3^\phi + 2c_4^\phi)}, \quad \Delta \omega = \omega_1 - M_{\text{BH}} \omega_{22}^{\text{mrg}} \]
\[ c_2^\phi = \alpha_{21}, \]

**Good performance of primary fits (modulo details...)**

Do this for various SXS dataset and then build up a (simple-minded) interpolating fit

Black-list:
1. the structure due to m<0 modes is not included (yet)
2. large-mass ratios/high spin: amplitude problems
3. problems are extreme for high-spin EMRL waves
4. more flexible fit-template needed
5. improve/check over all datasets (SXS & BAM for large mass-ratios & consistency with EMRL)
TEOBResumS point-mass potential

From EOB/NR-fitting:
\[ a_6^C(\nu) = 3097.3\nu^2 - 1330.6\nu + 81.38 \]

Years of analytical and numerical work to get this strong-field difference!

\[ \bar{F}(M) \equiv 1 - F = 1 - \max_{t_0,\phi_0} \frac{\langle h_{22}^{\text{EOB}}, h_{22}^{\text{NR}} \rangle}{\|h_{22}^{\text{EOB}}\| \|h_{22}^{\text{NR}}\|} \]
Spinning BBHs

Spin-orbit & spin-spin couplings

(i) Spins aligned with $L$: repulsive (slower) longer INSPIRAL

(ii) Spins anti-aligned with $L$: attractive (faster) shorter INSPIRAL

(iii) Misaligned spins: precession of the orbital plane (waveform modulation)

$$\chi_{1,2} = \frac{c S_{1,2}}{Gm_{1,2}^2}$$

EOB/NR agreement: sophisticated (though rather simple) model for spin-aligned binaries

Damour&Nagar, PRD90 (2014), 024054 (Hamiltonian)
Damour&Nagar, PRD90 (2014), 044018 (Ringdown)
Nagar,Damour, Reisswig & Pollney, PRD 93 (2016), 044046

AEI model, SEOBNRv4, Bohe et al., arXiv:1611.03703v1 (PRD in press)
Spin-Spin in Kerr Hamiltonian

Particle: \((\mu, S_*)\) \hspace{1cm} Kerr black-hole: \((M, S)\)

\[ H_{Kerr} = H_{orb}^{Kerr} + H_{SO}^S(S) + H_{SO}^{S*}(S_*) \]

\[ H_{orb,eq}(r, p_r, p_\varphi) = \sqrt{A_{eq}(r) \left( \mu^2 + \frac{p_\varphi^2}{r_c^2} + \frac{p_r^2}{B_{eq}(r)} \right)}. \]

\[ A_{eq}(r) = \frac{\Delta(r)}{r_c^2} = \left( 1 - \frac{2M}{r_c} \right) \frac{1 + \frac{2M}{r_c}}{1 + \frac{2M}{r}} \]

Centrifugal radius

\[ r_c^2 = r^2 + a^2 + \frac{2Ma^2}{r} \]

EOB: Identify a similar centrifugal radius in the comparable mass case and devise a similar deformation of \(A\)

Similar, though different, in SEOB
The effective Hamiltonian

\[ \hat{H}_{\text{eff}} = \frac{g_S^{\text{eff}}}{r^3} \mathbf{L} \cdot \mathbf{S} + \frac{g_{S^*}^{\text{eff}}}{r^3} \mathbf{L} \cdot \mathbf{S}^* + \sqrt{A(1 + \gamma^{ij} p_i p_j + Q_4(p))} \]

with the structure

\[ g_S^{\text{eff}} = 2 + \nu(\text{PN corrections}) + (\text{spin})^2 \text{corrections} \]
\[ g_{S^*}^{\text{eff}} = \left( \frac{3}{2} + \text{test mass coupling} \right) + \nu(\text{PN corrections}) + (\text{spin})^2 \text{corrections} \]
\[ A = 1 - \frac{2}{r} + \nu(\text{PN corrections}) + (\text{spin})^2 \text{corrections} \]
\[ \gamma^{ij} = \gamma^{ij}_{\text{Kerr}} + \nu(\text{PN corrections}) + \ldots \]

\[ \mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 = M^2(X_1^2 \chi_1 + X_2^2 \chi_2) \quad X_i = m_i/M \]
\[ \mathbf{S}^* = \frac{m_2}{m_1} \mathbf{S}_1 + \frac{m_1}{m_2} \mathbf{S}_2 = M^2 \nu(\chi_1 + \chi_2) \quad -1 \leq \chi_i \leq 1 \]
THE TWO TYPES OF SPIN-ORBIT COUPLINGS

\[ \hat{H}_{SO}^{\text{eff}} = G_S \mathbf{L} \cdot \mathbf{S} + G_S^* \mathbf{L} \cdot \mathbf{S}^* \]

\[ G_S = \frac{1}{r^3} g_S^{\text{eff}}, \quad G_S^* = \frac{1}{r^3} g_{S^*}^{\text{eff}} \]

In the Kerr limit, only \textit{S-type gyro-gravitomagnetic ratio} enters:

\[ g_S^{\text{eff}} = 2 \frac{r^2}{r^2 + a^2 \left( (1 - \cos^2 \theta) \left( 1 + \frac{2}{r} \right) + 2 \cos^2 \theta \right) + \frac{a^4}{r^2} \cos^2 \theta} = 2 + \mathcal{O}(\text{spin})^2 \]

PN calculations yield (in some spin gauge)\cite{DJS08, Hartung&Steinhoff11, Nagar11, Barausse&Buonanno11}

\[ g_S^{\text{eff}} = 2 + \frac{1}{c^2} \left\{ -\frac{15}{8} \nu - \frac{33}{8} (\mathbf{n} \cdot \mathbf{p})^2 \right\} + \frac{1}{c^4} \left\{ -\frac{1}{r^2} \left( \frac{51}{4} \nu + \frac{\nu^2}{8} \right) + \frac{1}{r} \left( -\frac{21}{2} \nu + \frac{23}{8} \nu^2 \right) (\mathbf{n} \cdot \mathbf{p})^2 + \frac{5}{8} \nu (1 + 7 \nu) (\mathbf{n} \cdot \mathbf{p})^4 \right\}, \quad + \frac{1}{c^6} \frac{\nu c_3}{r^3} \]

\[ g_{S^*}^{\text{eff}} = \frac{3}{2} + \frac{1}{c^2} \left\{ -\frac{1}{r} \left( \frac{9}{8} + \frac{3}{4} \nu \right) - \left( \frac{9}{4} \nu + \frac{15}{8} \right) (\mathbf{n} \cdot \mathbf{p})^2 \right\} + \frac{1}{c^4} \left\{ -\frac{1}{r^2} \left( \frac{27}{16} + \frac{39}{4} \nu + \frac{3}{16} \nu^2 \right) + \frac{1}{r} \left( \frac{69}{16} - \frac{9}{4} \nu + \frac{57}{16} \nu^2 \right) (\mathbf{n} \cdot \mathbf{p})^2 + \left( \frac{35}{16} + \frac{5}{2} \nu + \frac{45}{16} \nu^2 \right) (\mathbf{n} \cdot \mathbf{p})^4 \right\} + \frac{1}{c^6} \frac{\nu c_3}{r^3} \]

"Effective" NNNLO SO-coupling

This functions are resummed taking their Taylor-inverse

The NR-informed effective parameter makes the spin-orbit coupling stronger or weaker with respect to the straight analytical prediction
Spin-Spin within TEOBResumS

Define a “centrifugal radius”: at LO reads

\[ r_c^2 = r^2 + \hat{a}_0^2 \left( 1 + \frac{2}{r} \right) \]

where:

\[ \hat{a}_0^2 = \tilde{a}_1^2 + 2\tilde{a}_1\tilde{a}_2 + \tilde{a}_2^2 \]

or

\[ \hat{a}_0^2 = C_{Q1}(\tilde{a}_1)^2 + 2\tilde{a}_1\tilde{a}_2 + C_{Q2}(\tilde{a}_2)^2 \]

\[ A_{eq}(\nu, \chi_1, \chi_2) = A_{orb}^{\text{EOB}}(\nu, \kappa, r) \frac{1 + \frac{2}{r}}{1 + \frac{r}{2}} \]

\[ r_c^2 = r^2 + \hat{a}_0^2 \left( 1 + \frac{2}{r} \right) + \delta\hat{a}^2, \]

is the BH case. In general, from I-Love-Q [Yagi-Yunes]

One verifies that once plugged in the EOB Hamiltonian and re-expanded one obtains the standard PN Hamiltonian @LO [e.g., cf. Levi-Steinhoff]

Similarly one can act on LO SS terms in the waveform & flux

\[ C_Q = 1 \]
TEOBResumS: spin-aligned + tides

- Spin-orbit parameter informed by 30 BBH NR simulations
- BEST faithfulness with all NR available (200 simulations)
- Robust and simple
- Tides and spin-induced moment included (BNS)
- ONLY publicly available stand-alone EOB code

\[
\tilde{F}(M) \equiv 1 - F = 1 - \max_{\tilde{\iota}_0, \phi_0} \frac{\langle h_{22}^{\text{EOB}}, h_{22}^{\text{NR}} \rangle}{\|h_{22}^{\text{EOB}}\| \|h_{22}^{\text{NR}}\|},
\]

Nagar, Bernuzzi, Del Pozzo et al., PRD98.104052

Effective NNNLO spin-orbit “function”

\[
c_3(\tilde{\alpha}_A, \tilde{\alpha}_B, \nu) = p_0 \frac{1 + n_1 \tilde{\alpha}_0 + n_2 \tilde{\alpha}_0^2}{1 + d_1 \tilde{\alpha}_0}
+ (p_1 \nu + p_2 \nu^2 + p_3 \nu^3) \tilde{\alpha}_0 \sqrt{1 - 4\nu}
+ p_4 (\tilde{\alpha}_A - \tilde{\alpha}_B) \nu^2,
\]

(17)

\[
\tilde{\alpha}_{1,2} = X_{1,2} \chi_{1,2}
\]

\[
X_{1,2} \equiv \frac{m_{1,2}}{M}
\]

\[
\tilde{\alpha}_0 \equiv \frac{S + S_*}{M^2} = X_A \chi_A + X_B \chi_B = \tilde{\alpha}_A + \tilde{\alpha}_B
\]

ONLY 2 EOBNR models

TEOBResumS

SEOBNRv4 (AEI)

See Rettegno, Martinetti, Nagar+2019, arXiv:1911.10818
TEOBResumS + Post Adiabatic Approx

ODEs are slow: 1-2s for BNS waveforms (10Hz) not good for DA
Shared solution: ROMs (surrogate models. Fast but not flexible)
Are ROMs really needed?

EOB equations of motion

\[
\frac{d\varphi}{dt} = \frac{1}{\nu \hat{H}_{\text{EOB}} \hat{H}_{\text{eff}}^{\text{orb}}} \left[ A \frac{p_\varphi}{r_c^2} + \hat{H}_{\text{eff}}^{\text{orb}} \tilde{G} \right],
\]  

(1)

\[
\frac{dr}{dt} = \left( \frac{A}{B} \right)^{1/2} \frac{1}{\nu \hat{H}_{\text{EOB}} \hat{H}_{\text{eff}}^{\text{orb}}} \times \\
\times \left[ p_{r*} \left( 1 + 2z_3 A \frac{p_{r*}^2}{r_c^2} \right) + \hat{H}_{\text{eff}}^{\text{orb}} p_\varphi \frac{\partial \tilde{G}}{\partial p_{r*}} \right],
\]  

(2)

\[
\frac{dp_\varphi}{dt} = \hat{F}_\varphi,
\]  

(3)

\[
\frac{dp_{r*}}{dt} = - \left( \frac{A}{B} \right)^{1/2} \frac{1}{2\nu \hat{H}_{\text{EOB}} \hat{H}_{\text{eff}}^{\text{orb}}} \left[ A' + p_\varphi^2 \left( \frac{A}{r_c^2} \right)' + \\
+ z_3 p_{r*}^4 \left( \frac{A}{r_c^2} \right)' + 2 \hat{H}_{\text{eff}}^{\text{orb}} p_\varphi \tilde{G}' \right],
\]  

(4)

\[
\tilde{G} \equiv G_S S + G_{S*} S_*
\]
TEOBResumS + Post Adiabatic Approx

Post-adiabatic approximation (Damour & AN, 2007)
2PA: used to have eccentricity free ID for the EOB EoM

\[ \hat{F}_\varphi(r) = \sum_{n=0}^{\infty} F_{2n+1}(r) \varepsilon^{2n+1} \]
\[ p_\varphi^2(r) = j_0^2(r) \left( 1 + \sum_{n=1}^{\infty} k_{2n}(r) \varepsilon^{2n} \right) \]
\[ p_{r*}(r) = \sum_{n=0}^{\infty} \pi_{2n+1}(r) \varepsilon^{2n+1} \]

Iterate up to nth order at a given radius to obtain the momenta with high accuracy
[Nagar&Rettegno, 2018]
\begin{align*}
    t &= \int_{r_{\text{max}}}^{r} dr \left( \hat{\partial}_{p_r} \hat{H} \right)^{-1} \\
    \varphi &= \int_{0}^{t} dt \hat{\partial}_{p_\varphi} \hat{H} = \int_{r_{\text{max}}}^{r} dr \hat{\partial}_{p_\varphi} \hat{H} \left( \hat{\partial}_{p_r} \hat{H} \right)^{-1}
\end{align*}

FIG. 1. Waveform comparison, $\ell = m = 2$ strain mode: EOB$_{\text{PA}}$ inspiral (colors) versus EOB inspiral obtained solving the ODEs (black). The orange vertical line marks the EOB LSO crossing for $(1, -0.99, -0.99)$ and $(3, +0.80, -0.20)$, while it corresponds to $r = 6$-crossing for $(1, +0.90, +0.90)$. The 4PA approximation already delivers an acceptable EOB/EOB$_{\text{PA}}$ agreement for both phase, $\phi$, and amplitude, $A$. This is improved further by the successive approximations. At 8PA, the GW phase difference is $\lesssim 10^{-3}$ rad up to $\sim 3$ orbits before merger. The light-gray curve also incorporates the EOB-merger and ringdown.
TABLE IV. Summary of the parameters that characterize GW150914 as found by cpnest and using TEOBResumS as template waveform, compared with the values found by the LVC collaboration [135]. We report the median value as well as the 90% credible interval. For the magnitude of the dimensionless spins $|\chi_A|$ and $|\chi_B|$ we also report the 90% upper bound. Note that we use the notation $\chi_{\text{eff}} \equiv \hat{a}_0$ for the effective spin, as introduced in Eq. (8).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>TEOBResumS</th>
<th>LVC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detector-frame total mass $M/M_\odot$</td>
<td>$73.6^{+5.7}_{-5.2}$</td>
<td>$70.6^{+4.6}_{-4.5}$</td>
</tr>
<tr>
<td>Detector-frame chirp mass $\mathcal{M}/M_\odot$</td>
<td>$31.8^{+2.6}_{-2.4}$</td>
<td>$30.4^{+2.1}_{-1.9}$</td>
</tr>
<tr>
<td>Detector-frame remnant mass $M_f/M_\odot$</td>
<td>$70.0^{+5.0}_{-4.6}$</td>
<td>$67.4^{+4.1}_{-4.0}$</td>
</tr>
<tr>
<td>Magnitude of remnant spin $\hat{a}_f$</td>
<td>$0.71^{+0.05}_{-0.07}$</td>
<td>$0.67^{+0.05}_{-0.07}$</td>
</tr>
<tr>
<td>Detector-frame primary mass $M_A/M_\odot$</td>
<td>$40.2^{+5.1}_{-3.7}$</td>
<td>$38.9^{+5.6}_{-4.3}$</td>
</tr>
<tr>
<td>Detector-frame secondary mass $M_B/M_\odot$</td>
<td>$33.5^{+4.0}_{-5.5}$</td>
<td>$31.6^{+4.2}_{-4.7}$</td>
</tr>
<tr>
<td>Mass ratio $M_B/M_A$</td>
<td>$0.8^{+0.1}_{-0.2}$</td>
<td>$0.82^{+0.20}_{-0.17}$</td>
</tr>
<tr>
<td>Orbital component of primary spin $\chi_A$</td>
<td>$0.2^{+0.6}_{-0.8}$</td>
<td>$0.39^{+0.49}_{-0.29}$</td>
</tr>
<tr>
<td>Orbital component of secondary spin $\chi_B$</td>
<td>$0.0^{+0.9}_{-0.8}$</td>
<td>$0.44^{+0.50}_{-0.40}$</td>
</tr>
<tr>
<td>Effective aligned spin $\chi_{\text{eff}}$</td>
<td>$0.1^{+0.1}_{-0.2}$</td>
<td>$-0.07^{+0.16}_{-0.17}$</td>
</tr>
<tr>
<td>Magnitude of primary spin $</td>
<td>\chi_A</td>
<td>$</td>
</tr>
<tr>
<td>Magnitude of secondary spin $</td>
<td>\chi_B</td>
<td>$</td>
</tr>
<tr>
<td>Luminosity distance $d_L$/Mpc</td>
<td>$479^{+188}_{-235}$</td>
<td>$410^{+160}_{-180}$</td>
</tr>
</tbody>
</table>

Nagar, Bernuzzi, Del Pozzo et al., PRD, arXiv:1806.01772
**TEOBResumS on GW150914**

State of the art:
TEOBResumS: PA approx + ODE+LAL implementation
Neutron stars: tides & spin

**TEOBResumS today** [AN+, PRD98, 2018,104052]

- tidal effects + nonlinear-in-spin-effects ($S^2$, $S^3$, $S^4$, …) [AN+, PRD99, 2019,044007]
- analytically very complete model (almost final)
- $l=3$ GSF-informed + gravitomagnetic tides [Akcay+, PRD, 2019, in press]
- checked with (state-of-the-art but short) NR simulations up to merger
- EFFICIENT due to the post-adiabatic approximation [AN & Rettegno PRD99, 2019 021501]
- no precession (yet!)

<table>
<thead>
<tr>
<th>$f_0$ [Hz]</th>
<th>$r_0$</th>
<th>$r_{min}$</th>
<th>$N_r$</th>
<th>$\Delta r$</th>
<th>$\tau_{SPA}$ [sec]</th>
<th>$\tau_{ODE}$ [sec]</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>112.81</td>
<td>12</td>
<td>500</td>
<td>0.20</td>
<td>0.03</td>
<td>0.53</td>
</tr>
<tr>
<td>10</td>
<td>179.02</td>
<td>12</td>
<td>830</td>
<td>0.20</td>
<td>0.05</td>
<td>1.1</td>
</tr>
</tbody>
</table>

No real need of EOB-surrogate!
### TEOBResumS vs NR: BNS

<table>
<thead>
<tr>
<th>name</th>
<th>EOS</th>
<th>$M_{A,B}[M_\odot]$</th>
<th>$C_{A,B}$</th>
<th>$k_2^{A,B}$</th>
<th>$\kappa^T$</th>
<th>$\Lambda_2^{A,B}$</th>
<th>$\chi_{A,B}$</th>
<th>$C_{Q_A,Q_B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BAM:0095</td>
<td>SLy</td>
<td>1.35</td>
<td>0.17</td>
<td>0.093</td>
<td>73.51</td>
<td>392</td>
<td>0.0</td>
<td>5.491</td>
</tr>
<tr>
<td>BAM:0039</td>
<td>H4</td>
<td>1.37</td>
<td>0.149</td>
<td>0.114</td>
<td>191.34</td>
<td>1020.5</td>
<td>0.141</td>
<td>7.396</td>
</tr>
<tr>
<td>BAM:0064</td>
<td>MS1b</td>
<td>1.35</td>
<td>0.142</td>
<td>0.134</td>
<td>289.67</td>
<td>1545</td>
<td>0.0</td>
<td>8.396</td>
</tr>
</tbody>
</table>
GW170817- Parameter Estimation (LVC)

- Only existing EOB model independent from existing waveform models in LIGO/Virgo
- PE of the binary neutron star GW170817: arXiv:1811.12907 (GWTC-1)

\[ \tilde{\Lambda} = \frac{16}{13} \frac{(m_1 + 12m_2)m_1^4\Lambda_1 + (m_2 + 12m_1)m_2^4\Lambda_2}{M^5}. \]
Recent development

Improved spin content in fluxes
More robust resummation of waveform amplitudes

32 simulations to determine $c_3$
tested over $132+338$ waveforms
Do we trust NR?

(1.50, +0.50, +0.50)

$\Delta \phi_{22}^\text{EOBNR}$

$\Delta A_{22}^\text{EOBNR}/A_{22}^\text{NR}$

$\Re[\Psi_{22}]/\nu$

NR
EOB

$u/M \times 10^4$

$u/M \times 10^5$
Conclusions

SEOB vs TEOB: consistent BUT different. Analytic differences are being spelled out (MNR, in prep. 2019)
Spin sector very different!

TEOB is more efficient due to PA approx. Long inspirals. No need of surrogate (e.g., is being used on BNS GW190426)

Good analytic modeling needed for reducing systematics. All current GW signal are going to be re-analyzed with TEOB

BBH+higher modes (no spin): arXiv:1904.09550
Higher modes with spin: in progress

Next challenge: eccentricity (in progress)