Neutron stars as gravity laboratories

The gift of “universal” relations

George Pappas

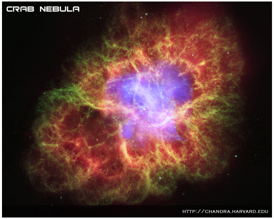
7 December 2019, Prague

MULTIMESSENGERS, COMPACT OBJECTS AND FUNDAMENTAL PHYSICS

ARISTOTLE UNIVERSITY OF THESSALONIKI

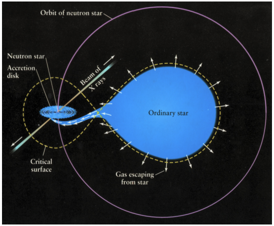
Neutron stars are the results of stellar evolution.
We can see them in stellar remnants.
A typical example is the Crab nebula that hosts the Crab pulsar[a].

[a] APOD 2006 October 26



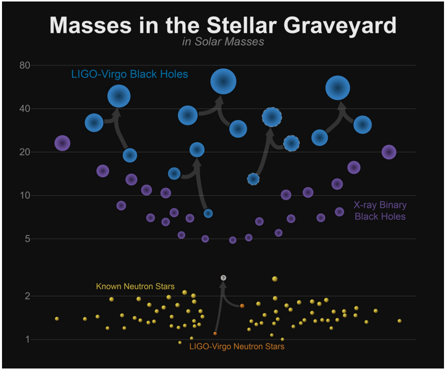
Very often we find rapidly rotating pulsars at the end of stellar evolution. The fastest rotating known pulsar (PSR J1748-2446ad) spins at 716Hz and it is part of a binary system[a].
Low mass X-ray binaries are systems that are comprised by a compact object (NS or BH) and a regular star companion. The main source of the X-rays is the accretion disk that forms around the compact object.

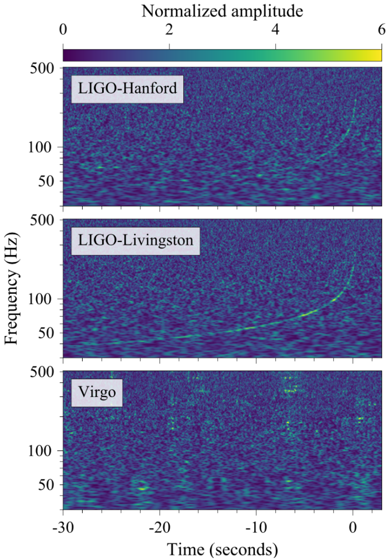
[a] J. W. T. Hessels et al., Science **311** 1901 (2006)



Interesting astrophysics takes place around NSs that depends on the background spacetime. Matter in their interior is at very high densities, where the equation of state is unknown. NSs have strong enough gravitational fields that can test our theories of gravity.

Furthermore with the detection of GW170817 we have "heard", through gravitational waves, the inspiral and collision of a binary neutron star system.[a]

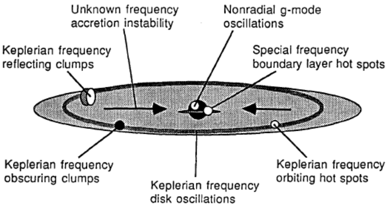




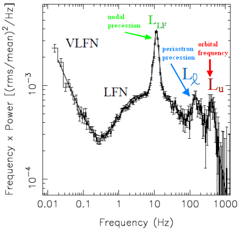
[a] B. P. Abbott et al.* (LIGO Scientific Collaboration and Virgo Collaboration), PRL **119**, 161101 (2017)

In low-mass X-ray binaries we can have observables related to geodesic motion. For example quasi-periodic oscillations of the spectrum[1] of an accretion disc.

Mechanisms for producing QPOs[2] from orbital motion

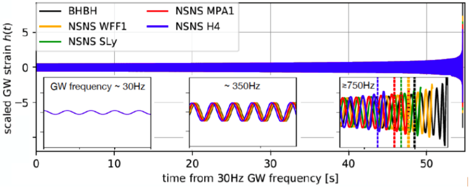


Typical X-Ray spectrum[3]



Such effects as well as NS binary inspirals probe the spacetime near the source.

Right: A. G. Chaves, T. Hinderer, J.Phys. G46 (2019) 123002



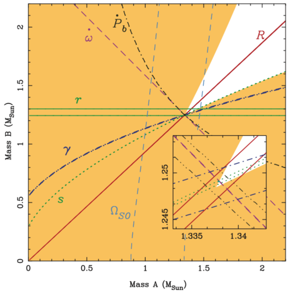
[1] Stella & Vietri, 1998, ApJ, 492, L59.

[2] F.K. Lamb, Advances in Space Research, 8 (1988) 421.

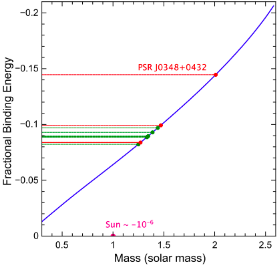
[3] Boutloukos et al., 2006, ApJ, 653, 1435-1444.

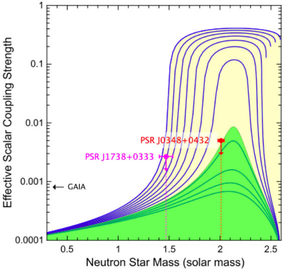
Pulsars in binary systems can be used to probe the spacetime around NSs and test GR and the structure of these objects.[4]

double pulsar: J0737-3039A,B



PSR J0348+0432



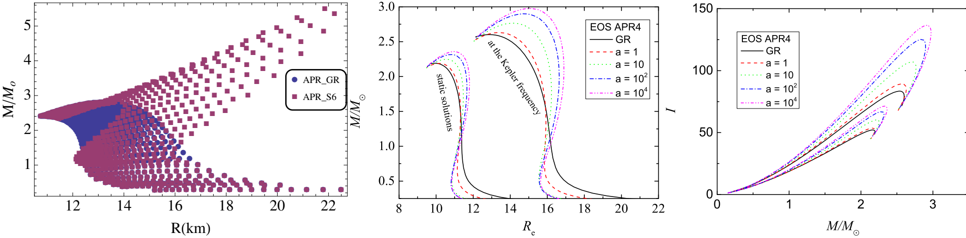


Paulo Freire has this covered

These systems though, are widely separated and can only probe the low PN order effects.

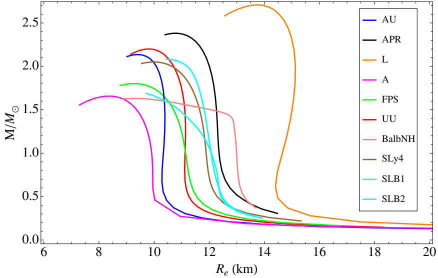
[4] Kramer et al., Science (2006); Antoniadis et al., Science (2013)

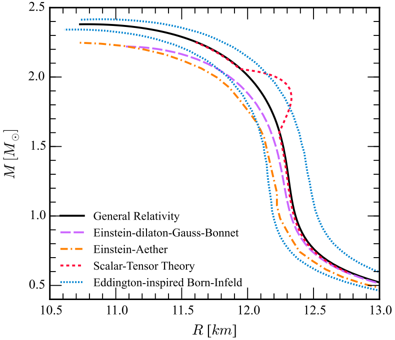
Neutron Stars in GR vs in alternative theories of gravity (Scalar-Tensor and R^2 gravity cases).



The structure of NSs is affected by modifications to GR and some times in a dramatic way (spontaneous scalarization).

There is generally a confusion problem between different EoSs within GR and a degeneracy problem between different EoSs within GR and modifications to GR[5] (while the EoS is unknown):





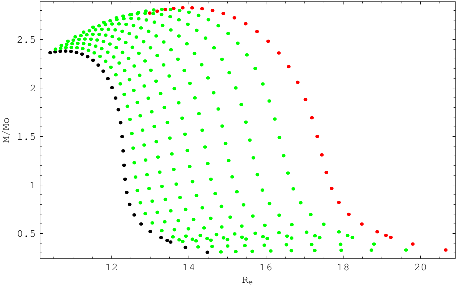
At this point, a description in terms of properties that are independent of the EoS would be very useful.
"Universal relations" are here to play that part.

[5] Kostas Glampedakis, GP, Hector O. Silva, Emanuele Berti, 2015 Phys. Rev. D 92, 024056.

Neutron Stars in GR

Results from numerical models:

Solving the GR field equations we can calculate models of rotating neutron stars for a given equation of state.



Rotating models for the APR EoS. The models with the fastest rotation have a spin parameter, $j = J/M^2$, around 0.7 and a ratio of the polar radius over the equatorial radius, r_p/r_e, around 0.56.

The numerical integration gives the various physical characteristics of the NS, the metric functions, as well as the relativistic multipole moments, i.e., M, $S_1 \equiv J$, $M_2 \equiv Q$, $S_3 \equiv J_3$ and M_4. [a] These moments characterise both NS and spacetime.

[a] G.P. & T. A. Apostolatos, Phys. Rev. Lett. **108** 231104 (2012); K. Yagi, K. Kyutoku, G. P., N. Yunes, & T.A. Apostolatos, Phys.Rev. D **89** 124013 (2014).

Neutron star multipole moments in GR

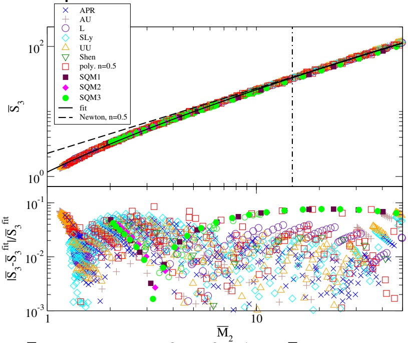
Black Hole-like behaviour of the moments:[6]

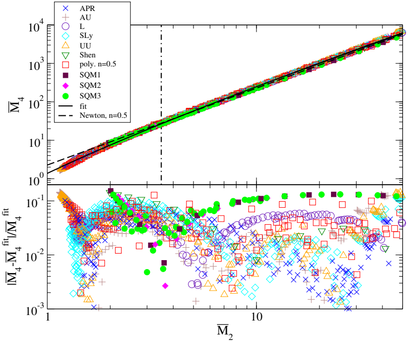
Kerr moments	Neutron star moments
$M_0 = M,$	$M_0 = M,$
$J_1 \equiv J = jM^2,$	$J_1 = jM^2,$
$M_2 \equiv Q = -j^2 M^3,$	$M_2 = -\alpha(EoS, \rho_c) j^2 M^3,$
$J_3 \equiv S_3 = -j^3 M^4,$	$J_3 = -\beta(EoS, \rho_c) j^3 M^4,$
$M_4 = j^4 M^5,$	$M_4 = \gamma(EoS, \rho_c) j^4 M^5,$
$\vdots$	$\vdots$
$M_{2n} = (-1)^n j^{2n} M^{2n+1},$	$M_{2n} = ?,$
$J_{2n+1} \equiv S_{2n+1} = (-1)^n j^{2n+1} M^{2n+2}$	$J_{2n+1} = ?$

where $j = J/M^2$.

[6] W.G. Laarakkers and E. Poisson, Astrophys. J. **512** 282 (1999); G.P. and T. A. Apostolatos, Phys. Rev. Lett. **108** 231104 (2012); K. Yagi, K. Kyutoku, G. P., N. Yunes, and T.A. Apostolatos, Phys.Rev. D **89** 124013 (2014).

EoS independent behaviour of the moments[7] :



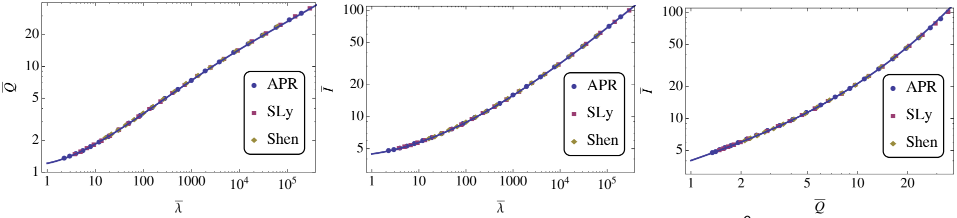


$\bar{M}_{2n} = |M_{2n}/(j^{2n}M^{2n+1})|$, $\bar{S}_{2n+1} = |S_{2n+1}/(j^{2n+1}M^{2n+2})|$

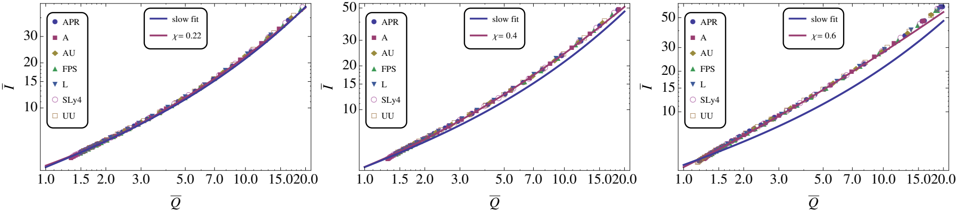
All these are properties that characterise the spacetime around neutron stars and are therefore relevant for doing astrophysics and are related to astrophysical observables.

[7] G.P. and T. A. Apostolatos, Phys.Rev.Lett. **112** 121101 (2014); K. Yagi, K. Kyutoku, G. P., N. Yunes, and T.A. Apostolatos, Phys.Rev. D **89** 124013 (2014).

Slow rotation Q-Love, I-Love and $I - Q$ relations[8] ($\bar{I} \equiv I/M^3$, $\bar{\lambda} \equiv \lambda/M^5$)



Slow and rapid rotation $I - Q$ relations ($\chi \equiv j$)[9]



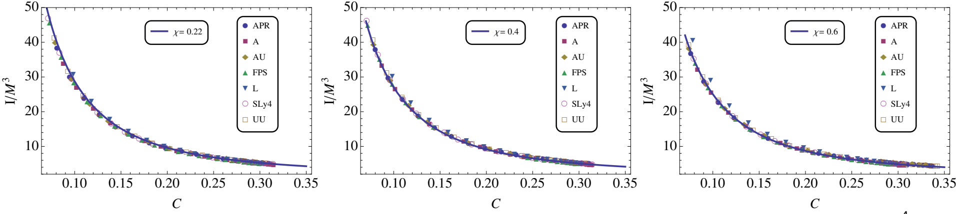
[8] K. Yagi and N. Yunes, Science 341, 365 (2013); Phys. Rev. D 88, 023009 (2013).

[9] Doneva et al., ApJ **781** L6 (2013); G.P. and T. A. Apostolatos, Phys.Rev.Lett. **112** 121101 (2014)

$I-\mathcal{C}$ relations:

First studied by Lattimer et al. and inspired by analytic models such as the Tolman VII model ($\rho=\rho_c[1-(r/R)^2]$).

$I-\mathcal{C}$ relations for different rotation rates[10]



$$I/M^3 = (1.471+0.448\chi) - \frac{0.0802+0.27289\chi}{\mathcal{C}} + \frac{0.438-0.0346\chi}{\mathcal{C}^2} - \frac{0.01694+0.0056\chi}{\mathcal{C}^3} + \frac{(3.316+1.57\chi)\times 10^{-4}}{\mathcal{C}^4},$$

where $\mathcal{C}=M/R$ is the compactness.

[10] C. Breu and L. Rezzolla, MNRAS 459, 646 (2016); K. V. Staykov, D. D. Doneva, and S. S. Yazadjiev, Phys. Rev. D 93, 084010 (2016); Doneva D.D., P.G. (2018) Universal Relations and Alternative Gravity Theories. In: Rezzolla L., Pizzochero P., Jones D., Rea N., Vidana I. (eds) The Physics and Astrophysics of Neutron Stars. Astrophysics and Space Science Library, vol 457. Springer, Cham.

Astro-seismology universal relations:[11]

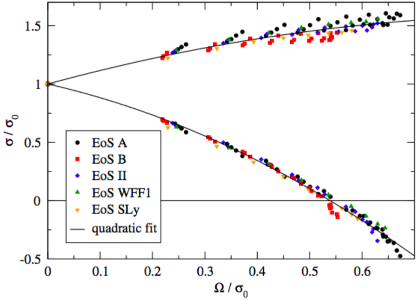


FIG. 1. Universal relations for the $l = |m| = 2$ f-mode frequencies from our polytropic models and two exemplary sequences based on realistic EoSs.

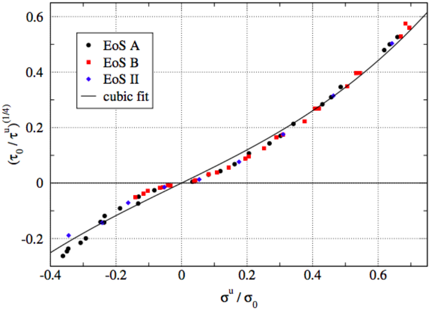
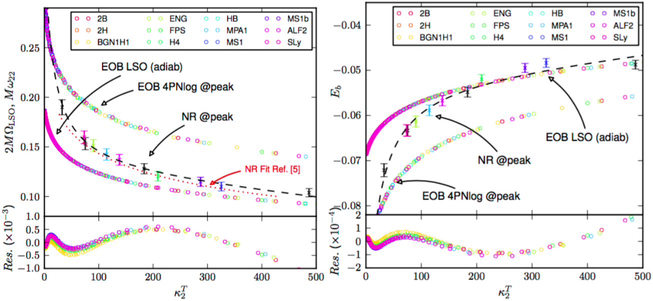


FIG. 2. Data points and universal relation for the damping time of the counter-rotating $l = m = 2$ f-mode.

Full result for rapidly rotating models without any approximations.

[11] Christian J. Krüger, Kostas D. Kokkotas, arXiv:1910.08370 [gr-qc]

Binary inspiral universal relations:

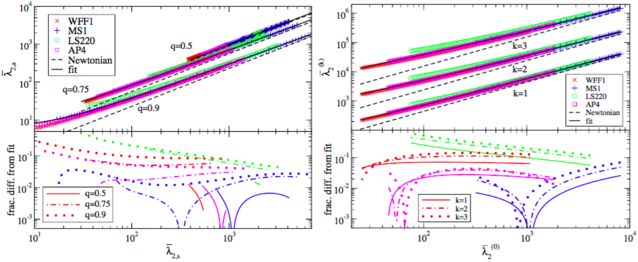


Peak GW frequency and binding energy vs tidal coupling.

Bernuzzi et al., Phys.Rev.Lett. 112 (2014) 201101.

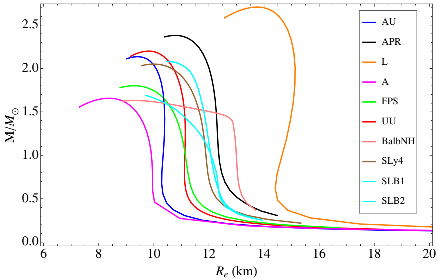
Binary Love Relations.

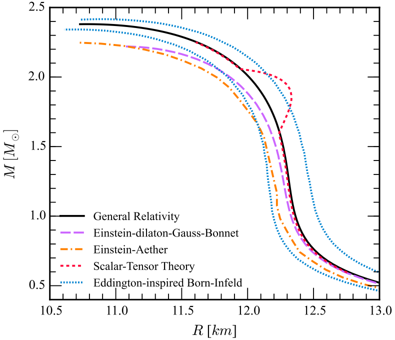
Yagi and Yunes, Phys. Rept. 681, 1 (2017).



Neutron Stars in other theories of gravity

Returning to NSs in modified theories of gravity, we were troubled by the confusion problem for the various EoSs in GR,





and this is also present in alternative theories of gravity as long as the EoS is unknown, while there is again the question of the degeneracy between different EoSs and different theories.

One could ask then, what happens with the Universal relations in alternative theories of gravity, and can they be used to break the degeneracies and tell theories apart?

In the case of Scalar-Tensor theories with a massless scalar field,

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}\left(\tilde{R} - 2\tilde{\nabla}^{\mu}\phi\tilde{\nabla}_{\mu}\phi\right) + S_m(g_{\mu\nu},\psi)\,,$$

the field equations in the Einstein frame take the form,

$$\tilde{R}_{ab} = 2\partial_a\phi\partial_b\phi + 8\pi G\left(T_{ab} - \frac{1}{2}g_{ab}T\right), \quad \tilde{g}^{ab}\tilde{\nabla}_a\tilde{\nabla}_b\phi = -4\pi\alpha(\phi)T$$

These equations can be solved as in GR in order to construct neutron stars.[12] On the other hand, the vacuum field equations can admit an Ernst formulation as in GR,[13]

$$(Re(\mathcal{E}))\nabla^2\mathcal{E} = \nabla\mathcal{E}\cdot\nabla\mathcal{E},$$

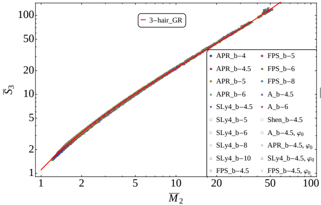
with the addition of a Laplace equation for the scalar field $\nabla^2\phi = 0$.
One can extend the definition of multipole moments in this case as well, where the moments (mass, spin, scalar) are defined in the Einstein frame. The actual physics though is done in the Jordan (physical) frame, where the metric is given by the conformal transformation $g_{\mu\nu} = A^2(\phi)\tilde{g}_{\mu\nu}$.[14]

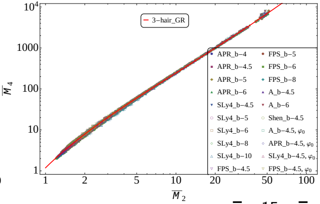
[12] D.D. Doneva, S.S. Yazadjiev, N. Stergioulas, K.D. Kokkotas, Phys. Rev. D 88, 084060 (2013)

[13] GP, T.P. Sotiriou, Phys. Rev. D91, 044011 (2015)

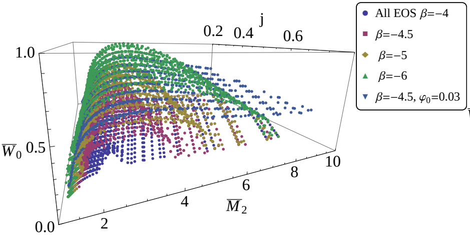
[14] GP, T.P. Sotiriou, MNRAS 454, 4066 (2015)

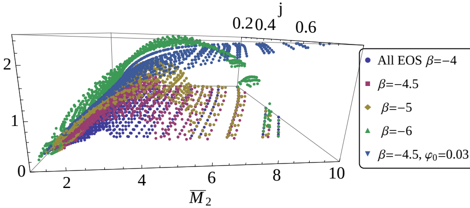
ST models against GR models and S_3^{ST}, M_4^{ST} vs Q^{ST} relations for various EoSs,





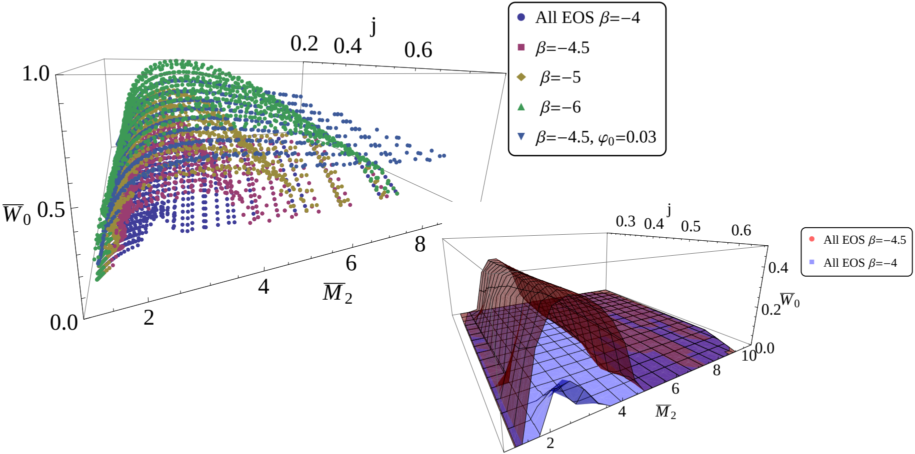
Scalar field normalised moments plotted against the spin j and $\bar{M}_2$,[15] $\bar{W}_a = f_\beta(j, \bar{M}_2)$





[15] G.P., D.D. Doneva, T.P. Sotiriou, S.S. Yazadjiev, and K.D. Kokkotas, Phys. Rev. D 99, 104014 (2019).

Scalar field normalised monopole plotted against the spin j and $\bar{M}_2$,[16]
$\bar{W}_0 = f_\beta(j, \bar{M}_2)$



---

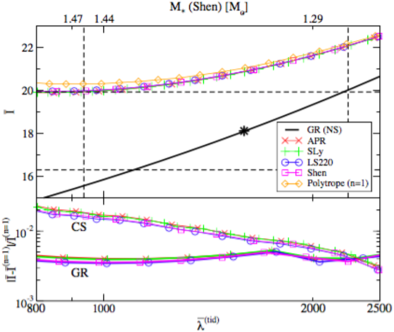
[16] G.P., D.D. Doneva, T.P. Sotiriou, S.S. Yazadjiev, and K.D. Kokkotas, Phys. Rev. D 99, 104014 (2019).

I-Love-Q relations:

Such relations have been examined in dCS gravity, in massless and massive scalar-tensor theories (ST), in Einstein-dilaton-Gauss-Bonnet (EdGB) gravity, in Eddington-inspired Born-Infeld (EiBI) theory and in $f(R)$ theories.

* The deviations from GR are almost negligible in the cases of the massless ST, the EdGB and the EiBI theory.

* Larger differences on the other hand are observed for dCS gravity, massive ST and $f(R)$ theories.

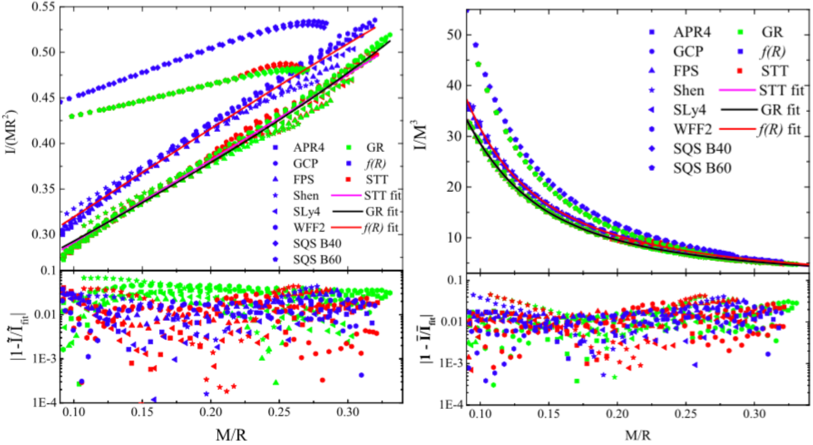


For example here we see how dCS compares to GR.[a]

[a] K. Yagi and N. Yunes, Phys. Rev. D 88, 023009 (2013).

$I - \mathcal{C}$ relations:

Slow rotation models for a massless ST theory, an $f(R) = R + \alpha R^2$ theory and GR (in ST $\beta = -4.5$).[17]



17 K. V. Staykov, D. D. Doneva, and S. S. Yazadjiev, Phys. Rev. D 93, 084010 (2016).

In the case of Scalar-Tensor theories with a massive scalar field,

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-\tilde{g}}\left(\tilde{R} - 2\tilde{\nabla}^{\mu}\phi\tilde{\nabla}_{\mu}\phi - 4V(\phi)\right) + S_m(g_{\mu\nu},\psi)\,,$$

the field equations in the Einstein frame take the form,

$$\tilde{R}_{ab} = 2\partial_a\phi\partial_b\phi + 8\pi G\left(T_{ab} - \frac{1}{2}g_{ab}T\right) + 2V(\phi)g_{ab}$$

$$\tilde{g}^{ab}\tilde{\nabla}_a\tilde{\nabla}_b\phi = -4\pi\alpha(\phi)T + \frac{dV(\phi)}{d\phi}$$

These equations can be solved as in GR in order to construct neutron stars.[18]

In this case, one cannot have a construction as in the massless case in order to define multipole moments as far as the scalar field is concerned. The mass and angular momentum moments though, far enough from the star, will be the same as in GR.

The question then is, how would the presence of the potential affect the properties and behaviour of the multipole moments in these theories. This is something to be explored in the near future.

[18] D.D. Doneva, S.S. Yazadjiev, N. Stergioulas, K.D. Kokkotas, Phys. Rev. D 88, 084060 (2013)

Using Universal Relations

A Rapidly Rotating Neutron Stars spacetime

Using the Weyl-Papapetrou line element for stationary and axisymmetric vacuum spacetimes,

$$ds^2 = -f\left(dt - \omega d\phi\right)^2 + f^{-1}\left[e^{2\zeta}\left(d\rho^2 + dz^2\right) + \rho^2 d\phi^2\right],$$

and the Ernst formulation of the Einstein field equations, one can construct spacetimes expressed in terms of the multipole moments,

$$f(\rho,z) = 1 - \frac{4M}{r_- + r_+ + 2M} + \frac{\alpha^2 j^4 M^6 \left(\rho^2 - 2z^2\right)^2}{2r^{10}} + \frac{2\beta j^4 M^6 z^2\left(2z^2 - 3\rho^2\right)}{r^{10}}$$
$$- \frac{\gamma j^4 M^5\left(3\rho^4 + 8z^4 - 24\rho^2 z^2\right)\left(r^2 - 2Mr\right)}{4r^{11}} - \frac{j^2 M^4}{14 r^{11}} A(M,\rho,z) + \frac{\alpha j^2 M^3}{7r^{11}} B(M,\rho,z),$$

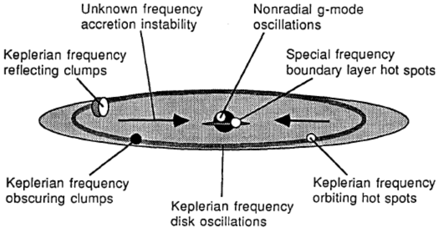
$$\omega(\rho,z) = -\frac{2jM^2\rho^2}{\left(\rho^2+z^2\right)^{3/2}} - \frac{2jM^3\rho^2}{\left(\rho^2+z^2\right)^2} + \frac{F(M,j,\beta,\rho,z)}{\left(\rho^2+z^2\right)^{7/2}} + \frac{H(M,j,\alpha,\beta,\rho,z)}{2\left(\rho^2+z^2\right)^4} + \frac{G(M,j,\alpha,\beta,\rho,z)}{4\left(\rho^2+z^2\right)^{11/2}},$$

$$\zeta(\rho,z) = \frac{1}{2}\log\left(\frac{r^2 - M^2 + r_- r_+}{2r_- r_+}\right) + \frac{j^2 M^4 \rho^2\left[(1-3\alpha)\rho^2 + 4(3\alpha-2)z^2\right]}{4r^8},$$

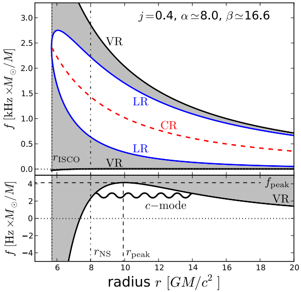
where $r_\pm = \sqrt{(M \pm z)^2 + \rho^2}$, $r = \sqrt{\rho^2 + z^2}$, and A, B, F, G, H, specific functions.

By prescribing values of β, γ following the 3-hair relations, the spacetime is a NS spacetime that is EoS independent[19] that one can use to do astrophysics.

[19] GP, (2017) MNRAS, 466, 4381; Maselli et al., arXiv:1905.05616 [astro-ph]



Geodesic and discoseismic models for NS QPOs.



Circular equatorial orbits:

$$\tilde{E} = \frac{-g_{tt} - g_{t\phi}\Omega}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega - g_{\phi\phi}\Omega^2}}, \quad \tilde{L} = \frac{g_{t\phi} + g_{\phi\phi}\Omega}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega - g_{\phi\phi}\Omega^2}}, \quad \Omega = \frac{-g_{t\phi,\rho} + \sqrt{(g_{t\phi,\rho})^2 - g_{tt,\rho}g_{\phi\phi,\rho}}}{g_{\phi\phi,\rho}}$$

For more general orbits: $-g_{\rho\rho}\left(\frac{d\rho}{d\tau}\right)^2 - g_{zz}\left(\frac{dz}{d\tau}\right)^2 = 1 - \frac{\tilde{E}^2 g_{\phi\phi} + 2\tilde{E}\tilde{L}g_{t\phi} + \tilde{L}^2 g_{tt}}{(g_{t\phi})^2 - g_{tt}g_{\phi\phi}} = V_{eff}$.

Perturbations give: $-g_{\rho\rho}\left(\frac{d(\delta\rho)}{d\tau}\right)^2 - g_{zz}\left(\frac{d(\delta z)}{d\tau}\right)^2 = \frac{1}{2}\frac{\partial^2 V_{eff}}{\partial\rho^2}(\delta\rho)^2 + \frac{1}{2}\frac{\partial^2 V_{eff}}{\partial z^2}(\delta z)^2$,

This equation describes the epicyclic frequencies, $\bar{\kappa}_\rho^2 = \frac{g^{\rho\rho}}{2}\frac{\partial^2 V_{eff}}{\partial\rho^2}\Big|_c$, $\bar{\kappa}_z^2 = \frac{g^{zz}}{2}\frac{\partial^2 V_{eff}}{\partial z^2}\Big|_c$.

From these we have, $\Omega_a = \Omega - \kappa_a$, i.e., the precession frequencies.

The precession frequencies are related to the spacetime multipole moments (Ryan, 1995),

in GR: $$\frac{\Omega_\rho}{\Omega} = 3U^2 - 4\frac{J_1}{M^2}U^3 + \left(\frac{9}{2} - \frac{3M_2}{2M^3}\right)U^4 - 10\frac{J_1}{M^2}U^5 + \left(\frac{27}{2} - 2\frac{J_1^2}{M^4} - \frac{21M_2}{2M^3}\right)U^6 + \dots$$
$$\frac{\Omega_z}{\Omega} = 2\frac{J_1}{M^2}U^3 + \frac{3M_2}{2M^3}U^4 + \left(7\frac{J_1^2}{M^4} + 3\frac{M_2}{M^3}\right)U^6 + \left(11\frac{J_1 M_2}{M^5} - 6\frac{S_3}{M^4}\right)U^7 + \dots$$

where $U = (M\Omega)^{1/3}$.
The Orbital frequency gives the Keplerian mass: $\Omega = (M/r^3)^{1/2}(1 + O(r^{-1/2}))$.

Having observations for the frequencies (such as QPOs) could be used to find the coefficients of the expansions.

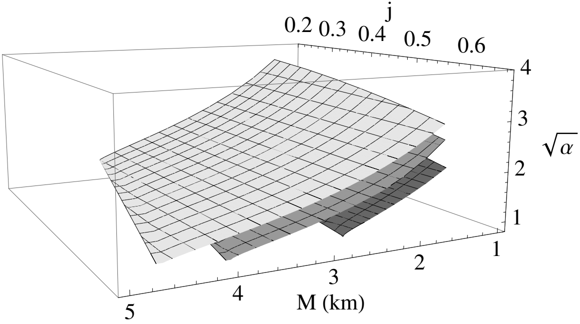
One could use the 3-hair universal relations to reduce the number of parameters and improve the fitting process.

Even better, one could use the EoS independent spacetime to model the various QPO frequencies and from there estimate the mass, angular momentum, and quadrupole of the NS, (M, j, α).[a]

[a] David Tsang and GP, (2016) ApJ, 818, L11; GP, (2017) MNRAS, 466, 4381; Maselli et al., arXiv:1905.05616 [astro-ph]

Moments and equation of state in GR

Determining the first 3 moments (M, J, M_2), i.e., (M, j, α), could be used to select an EoS[a] out of the realistic EoS candidates (fig. shows 3 different EoSs, where each EoS traces a surface in the parameter space).



[a] G.P. and T. A. Apostolatos, Phys.Rev.Lett. **112** 121101 (2014)

In Scalar-Tensor theory, the frequencies have a modified dependence on the moments.[20]

$$\frac{\Omega_\rho}{\Omega} = \left(3 - \frac{W_0\left(\beta_0 W_0 - 8\alpha_0 \bar{M}\right)}{2\bar{M}^2}\right) U^2 - \frac{4J_1}{\bar{M}^2} U^3 + \left[\left(\frac{9}{2} - \frac{3M_2}{2\bar{M}^3}\right) + (\beta_0 - 1)\frac{W_0^2}{\bar{M}^2} - \frac{13\beta_0^2 W_0^4}{24\bar{M}^4}\right] U^4 + \ldots$$

$$\frac{\Omega_z}{\Omega} = \frac{2J_1}{\bar{M}^2} U^3 + \frac{3(M_2 - \alpha_0 W_2)}{2\bar{M}^3} U^4 - \frac{2J_1 W_0\left(\beta_0 W_0 - \alpha_0 \bar{M}\right)}{\bar{M}^4} U^5 + \ldots$$

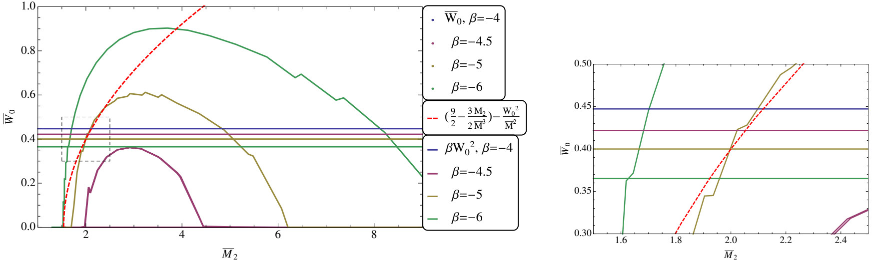
where $U = (\bar{M}\Omega)^{1/3}$. The calculations are done in the Jordan frame. Again the orbital frequency gives the Keplerian mass: $\Omega = (\bar{M}/r^3)^{1/2}(1 + O(r^{-1/2}))$, but this time the Keplerian mass is $\bar{M} = M - W_0\alpha_0$. W_0 is the scalar charge, W_2 is the scalar quadrupole and $\alpha_0 \equiv (d\ln A)/d\phi$, $\beta_0 \equiv d\alpha/d\phi$.

These observables could in principle distinguish between GR and Scalar-Tensor theory.

[20] GP, T.P. Sotiriou, MNRAS 454, 4066 (2015)

Moments and theory of gravity:

Radial precession frequency: $(\Omega_r/\Omega) = \sum C_a \Omega^{a/3}$, where we can imagine measuring the coefficients up to C_5^{STT}, using QPOs. These together with $\bar{W}_0 = f_\beta(j, \bar{M}_2)$ can give us β and W_0.[21]



The following onstraints between the coefficients C_5^{STT} give the above curves.

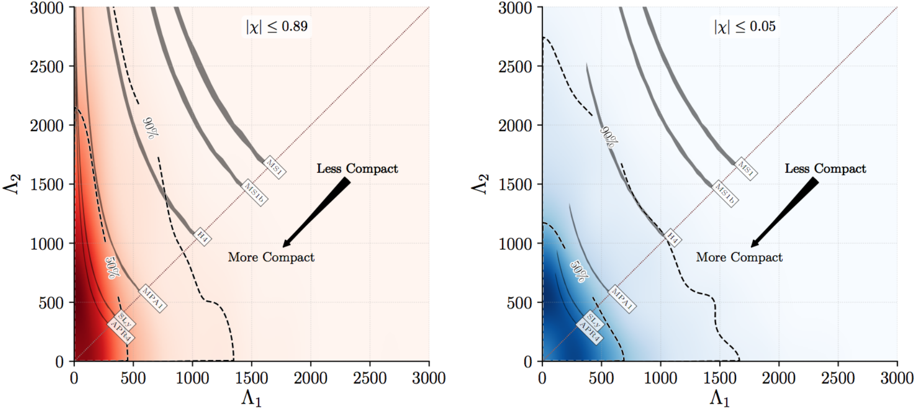
$$45 C_3^{\mathrm{STT}} \bar{M}^{2/3} = 10 C_2^{\mathrm{STT}} C_3^{\mathrm{STT}} + 6 C_5^{\mathrm{STT}}, \quad j = -\frac{C_3^{\mathrm{STT}}}{4\bar{M}}, \quad \beta\left(W_0/\bar{M}\right)^2 = 2\left(3 - C_2^{\mathrm{STT}} \bar{M}^{-2/3}\right),$$

$$\left[\left(\frac{9}{2} - \frac{3M_2}{2\bar{M}^3}\right) - \frac{W_0^2}{\bar{M}^2}\right] = \bar{C}_4.$$

[21] G.P., D.D. Doneva, T.P. Sotiriou, S.S. Yazadjiev, and K.D. Kokkotas, Phys. Rev. D 99, 104014 (2019).

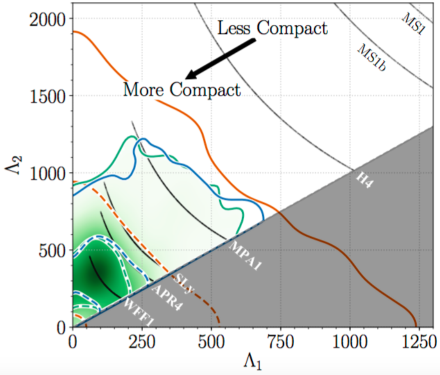
From the gravitational wave phase one can extract information for the Love numbers of each of the two stars, Λ_1 and Λ_2. In the fig. stars with larger radii are towards up and right. This gives an estimate of $R \lesssim 14km$ for an $1.4M_\odot$ NS.[a]

[a] B. P. Abbott et al.* (LIGO Scientific Collaboration and Virgo Collaboration), PRL **119**, 161101 (2017)



Using the universal relation between Λ_a, Λ_s and $q = m_2/m_1$ for the binary and the $\Lambda - C$ relation for an individual star, one can improve upon the previous results.[a]

[a] B. P. Abbott et al.* (LIGO Scientific Collaboration and Virgo Collaboration), PRL **121**, 161101 (2018)



In addition, the results form GW170817 have been used to reduce the variability of these relations, which will increase the accuracy of future measurements.[22]

[22] Carson et al., Phys.Rev. D99 (2019) 083016

- There are several NS properties that show universal behaviour (EoS independent).
- These universal properties are present in several alternative theories of gravity as well, such as in ST theories.
- Universal relations are already being used (in GW analysis) and,
- can be used to constrain the EoS of NSs.
- Some of these relations could also be used to distinguish between some alternative theories.
- Furthermore one can use universal relations to construct useful tools, such as an analytic universal NS spacetime, to do astrophysics with.

Thank you