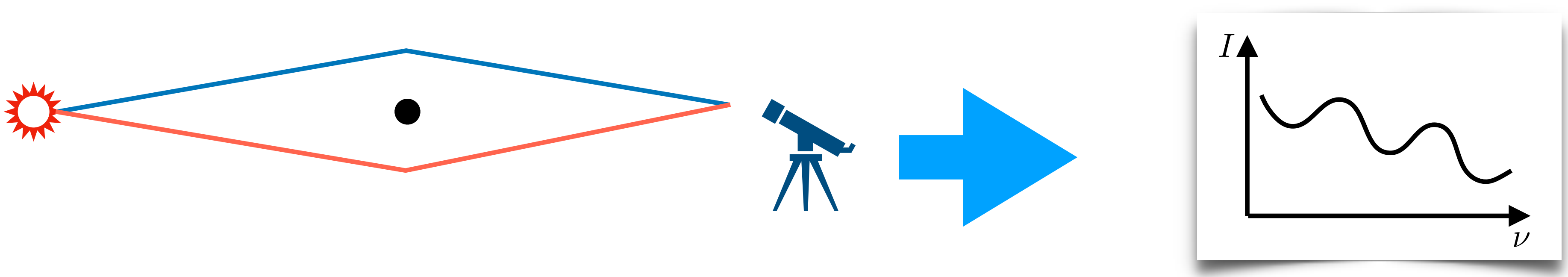
Probing dark matter with diffractive gravitational lensing of GRBs and FRBs

Sergey Sibiryakov

w/ Andrey Katz, Joachim Kopp and Wei Xue,

1807.11495, 1912.xxxxx



Multimessengers, Prague, 5/12/19

Which Dark Matter ?

MACHOs - MAssive Compact Halo Objects

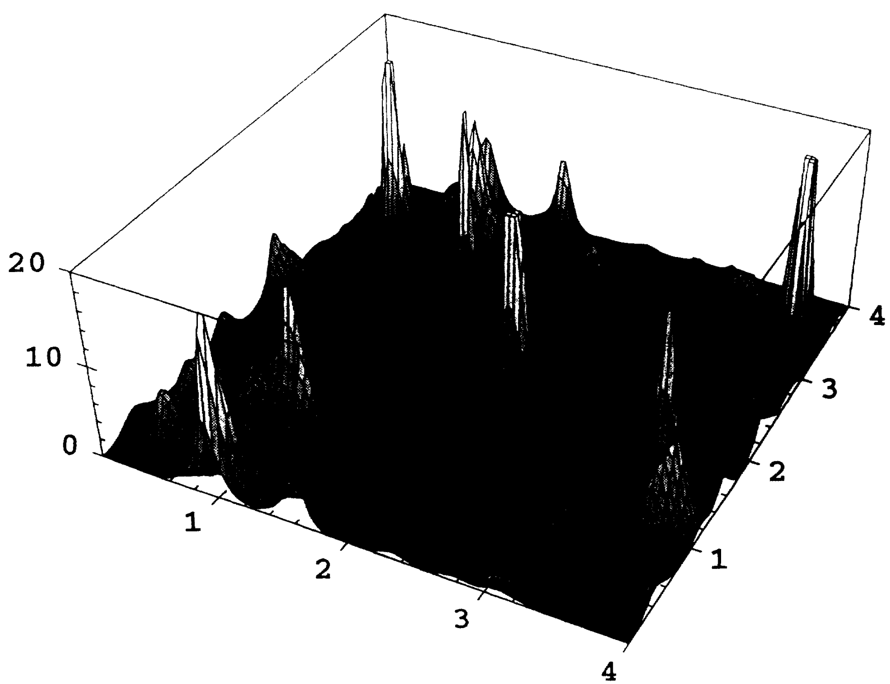
$$10^{-17} M_\odot < M < 10^3 M_\odot$$

Examples :

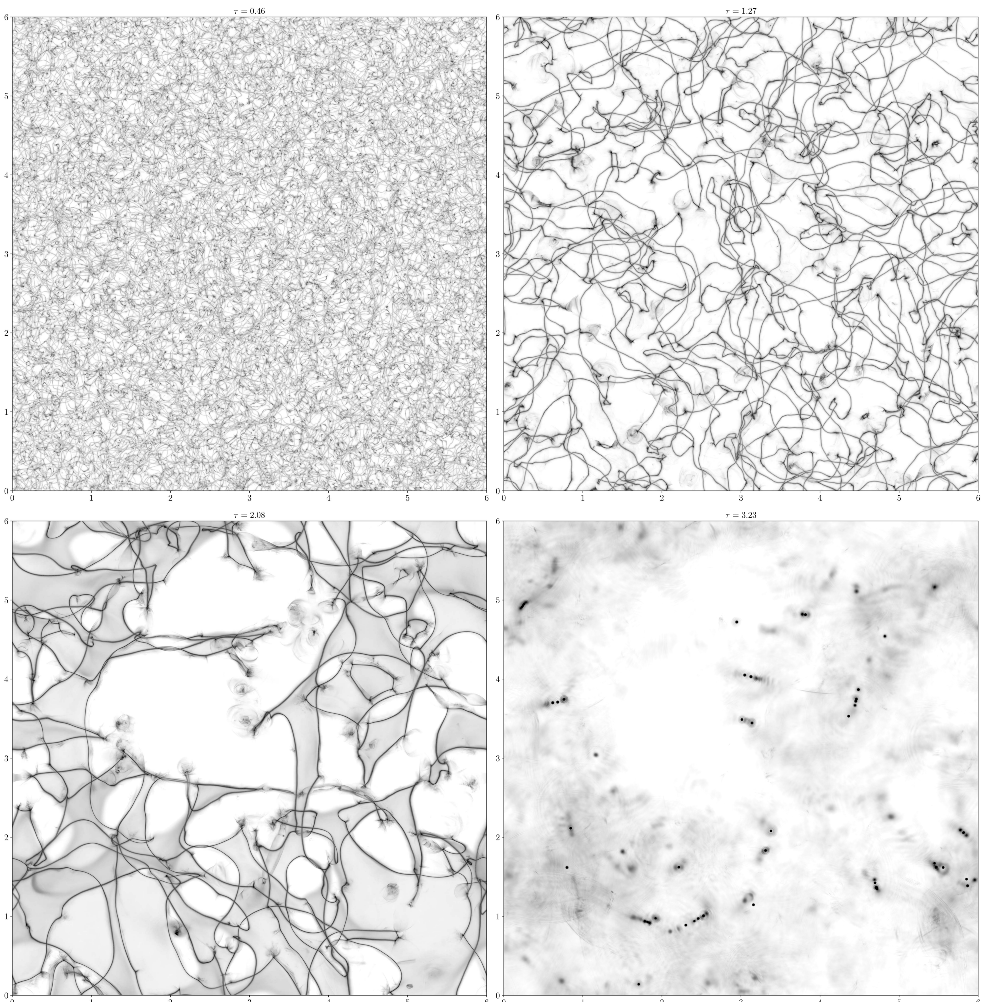
Carr & Hawking (1974)

- primordial black holes
- axion miniclusters / ultracompact minihalos

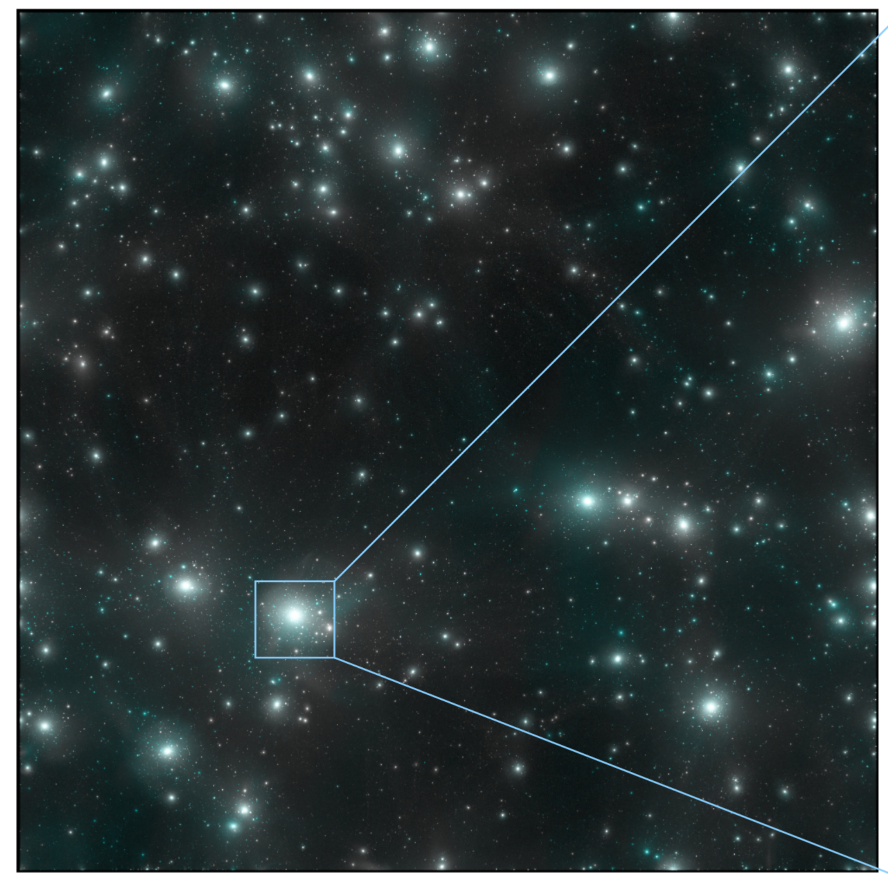
Hogan & Rees (1988)
Kolb & Tkachev (1993)



Vaquero, Redondo & Stadler (2018)

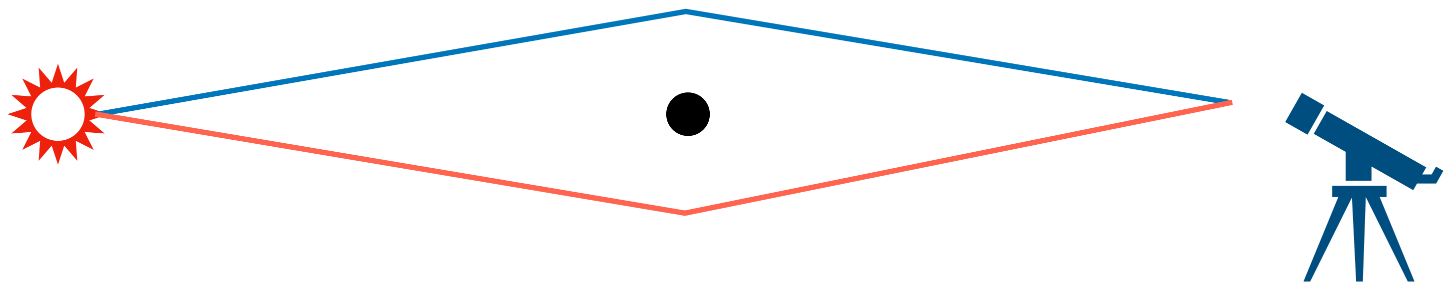


Eggemeier et al. (2019)



How to search ?

Gravitational lensing



Many surveys inside Milky Way / Local Group: MACHO, EROS, OGLE, SUBARU. Use microlensing technique: look for variations of observed intensity of background stars

Lensing at cosmological scales

Advantages :

- optical depth $n_{dm} r_E^2 D_S = 4G\rho_{dm} D_S^2 \propto (H_0 D_S)^2 \Omega_{dm}$

- DM mass function can be different in halos (e.g. miniclusters may be destroyed by tidal forces)

Challenges :

- the sources must be smaller than the Einstein ring = pointlike

→ Gamma Ray Bursts and Fast Radio Bursts

- angular separation between the images $\sim \mu as$ for lens of solar mass — too small to be resolved

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Diffractive lensing

Gould (1992)

Idea: interference between two unresloved images produces fringes in the frequency spectrum

lensing time delay

$$F_{\rm obs}(t) = AF_{in}(t) + BF_{\rm in}(t - \Delta t)$$

$$\Rightarrow \quad f_{\rm obs}(\omega) = Af_{in}(\omega) + Bf_{\rm in}(\omega)e^{i\omega\Delta t}$$

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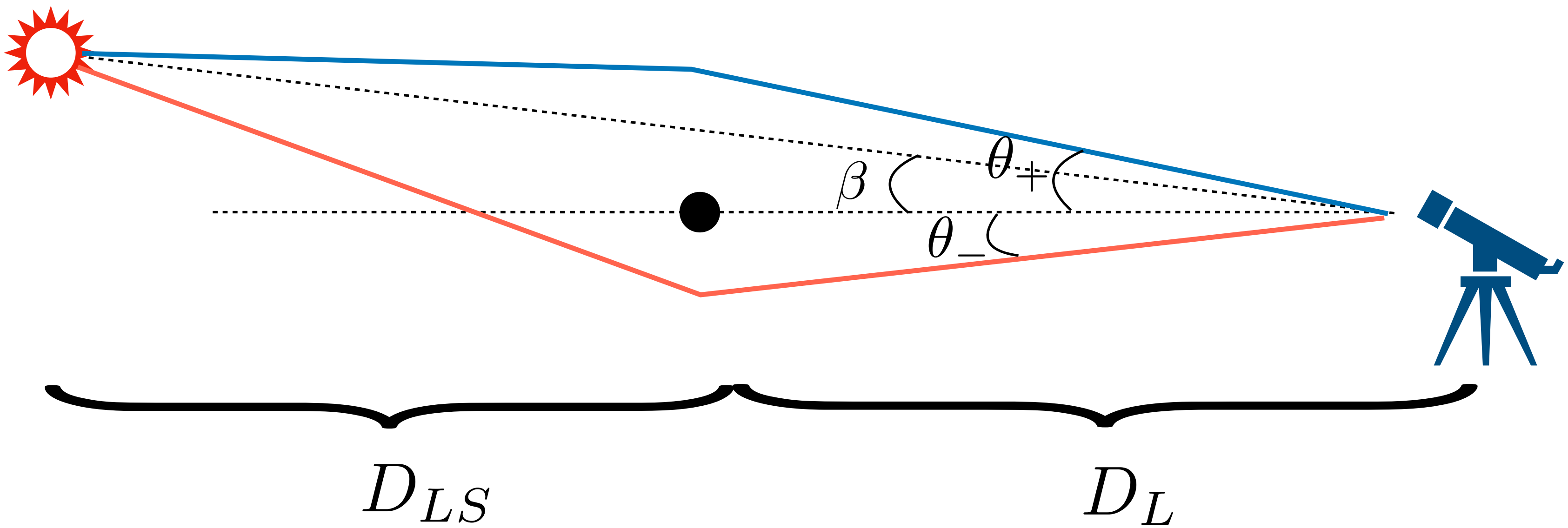
Lensing generalities



$$\Delta t = \frac{D_L D_S}{D_{LS}}(1+z_L)\left(\frac{|\vec{\theta}-\vec{\beta}|^2}{2} - \psi(\vec{\theta})\right)$$

lensing potential

$$\nabla^2\psi = 8\pi G\,\frac{D_{LS}D_L}{D_S}\int d\xi\,\rho\left(\sqrt{(D_L\theta)^2+\xi^2}\right)$$

$$\nabla(\Delta t) = 0 \Rightarrow \theta_\pm$$

Pointlike source and lens, geometric optics

$$\psi(\theta) = \theta_E^2 \log\theta$$

Einstein angle $\theta_E = \left(4GM\frac{D_{LS}}{D_S D_L}\right)^{1/2}$

$$\theta_\pm = \frac{1}{2}\left(\beta \pm \sqrt{\beta^2 + 4\theta_E^2}\right)$$

Magnifications: $\mu_\pm = \frac{y^2+2}{2y\sqrt{y^2+4}} \pm \frac{1}{2}$

Total magnification:

$$\mu = \frac{y^2+2}{y\sqrt{y^2+4}} + \frac{2}{y\sqrt{y^2+4}}\sin\left(\Omega\left[\frac{y\sqrt{y^2+4}}{2} + \log\left|\frac{y+\sqrt{y^2+4}}{y-\sqrt{y^2+4}}\right|\right]\right)$$

$$y = \beta/\theta_E \qquad \Omega = 4GM(1+z_L)\omega$$

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Extended lens

Power-law profile $\rho \propto r^{-\delta}$ $\Rightarrow$ $\psi \propto \theta^{3-\delta}$

Lens equation: $\theta_{\pm} - \theta_0^{\delta-1}\theta_{\pm}^{2-\delta} = \beta$

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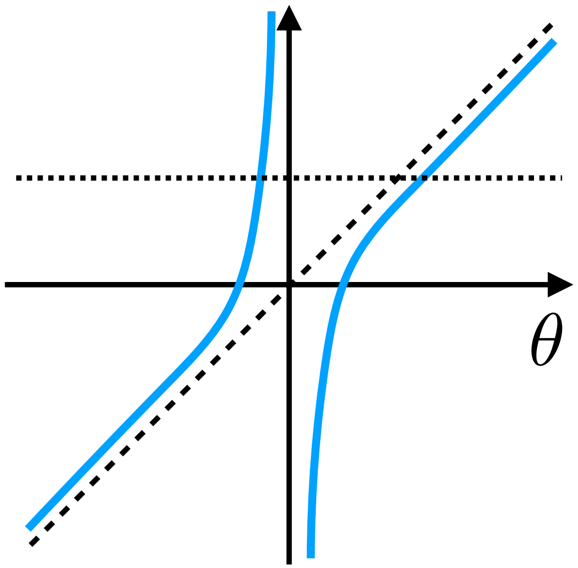
$\delta > 2$



two images

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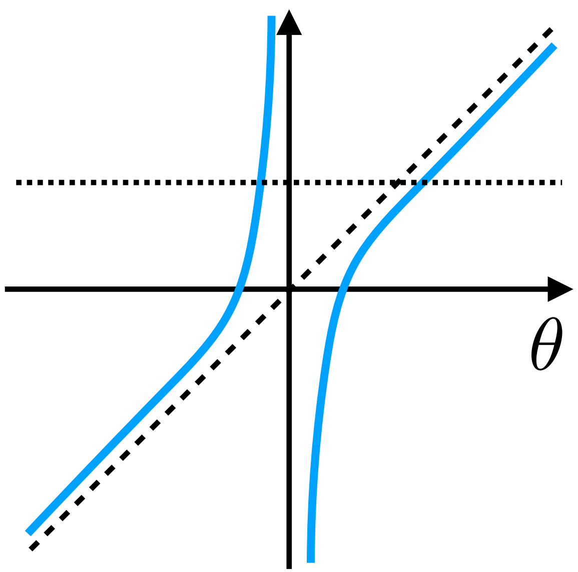
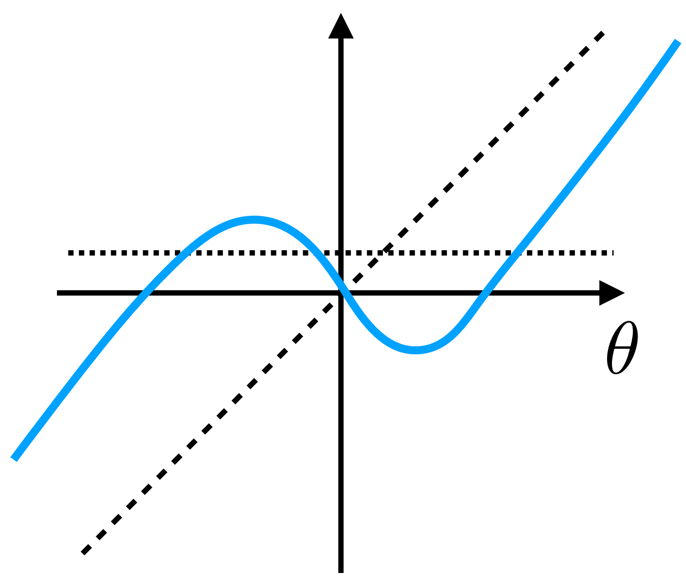


two images

$2 > \delta > 1$



three or one image

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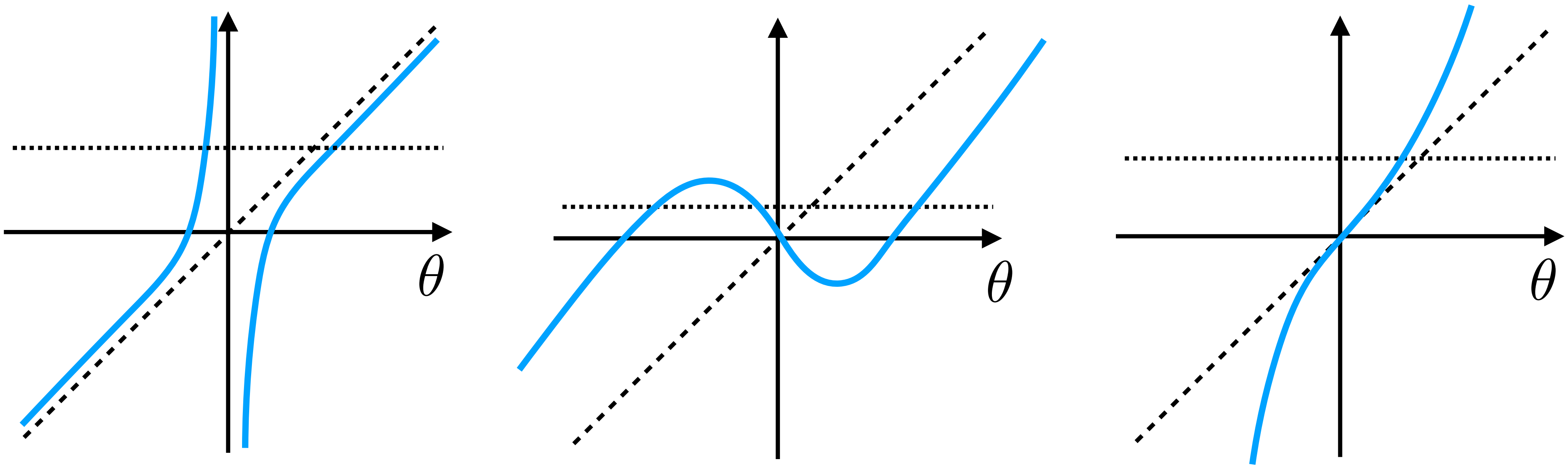
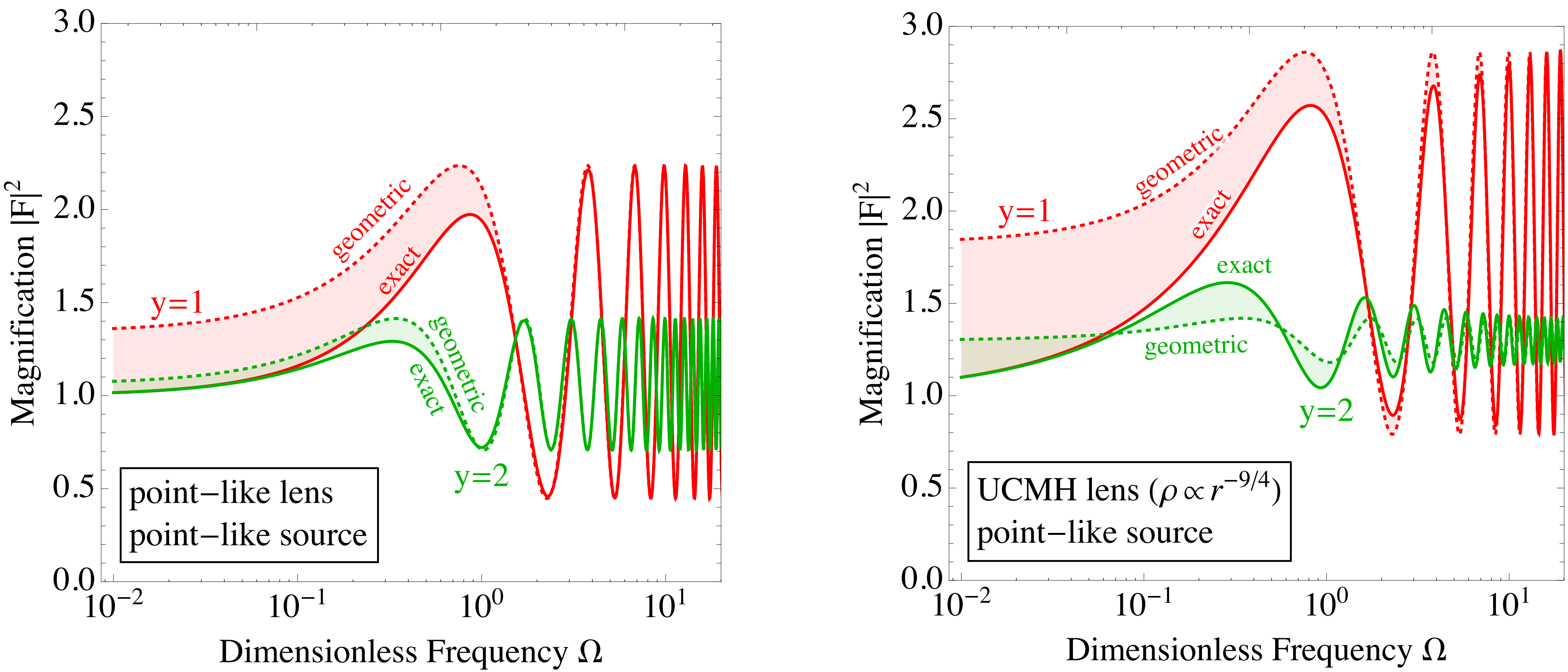
$\delta > 2$ $\quad$ $2 > \delta > 1$ $\quad$ $1 > \delta$



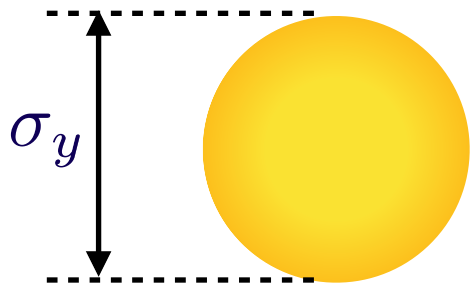
two images $\quad$ three or one image $\quad$ single image

Geometric vs. wave optics

Exact magnification: $\mu = \left| \frac{\Omega}{(2\pi i)\theta_E^2} \int d^2\vec{\theta}\, e^{i\omega \Delta t(\vec{\theta})} \right|^2$



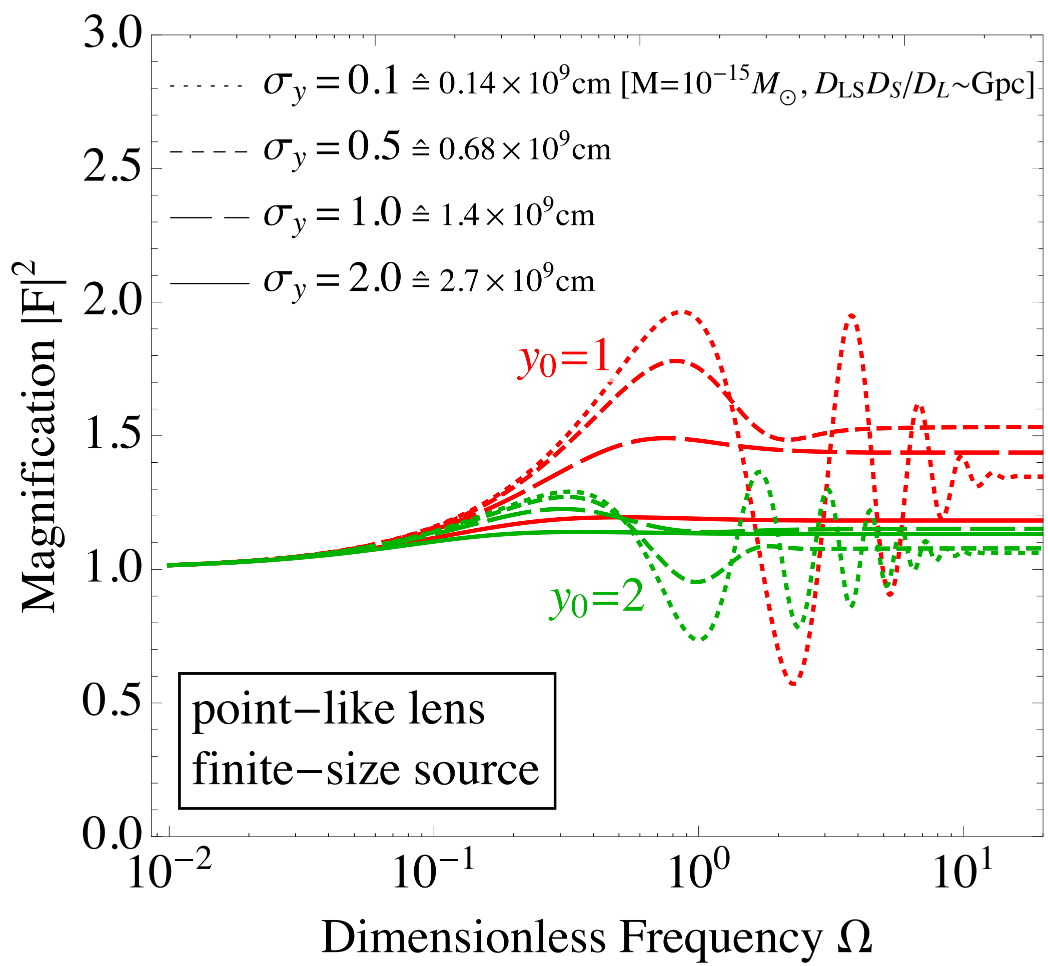
Extended source



Intensity profile $W(\vec{y};\sigma_y)$

Fringes average out:

$$\bar{\mu} = \frac{\int d^2y\, W(\vec{y};\sigma_y)\,\mu(\vec{y};\Omega)}{\int d^2y\, W(\vec{y};\sigma_y)}$$



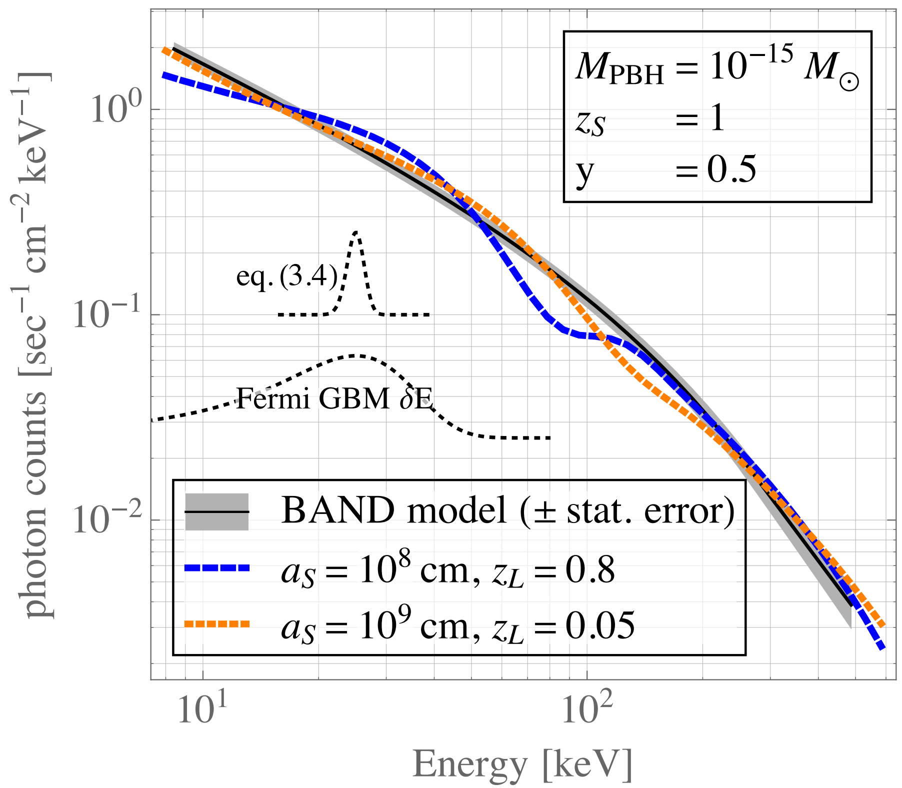
Application to GRBs (femtolensing)

Gould (1992)
Stanek, Paczynski, Goodman (1993)

Duration Short GRBs from 0.1 to 2 s, Long GRBs from 2 to 200 s

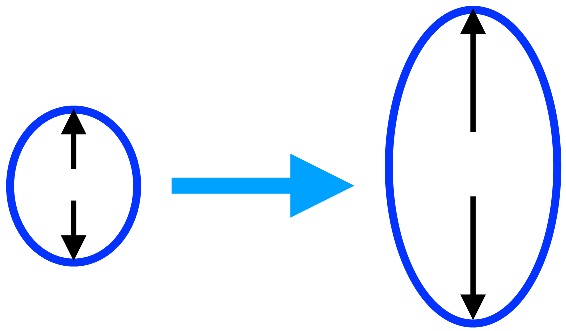
Broad spectrum from 10 keV to 10 MeV

$E\Delta t \sim 1$ potentially sensitive to $10^{-17} M_\odot < M < 10^{-14} M_\odot$



Are GRBs compact enough ?

Transverse size can be estimated from the variability time scale



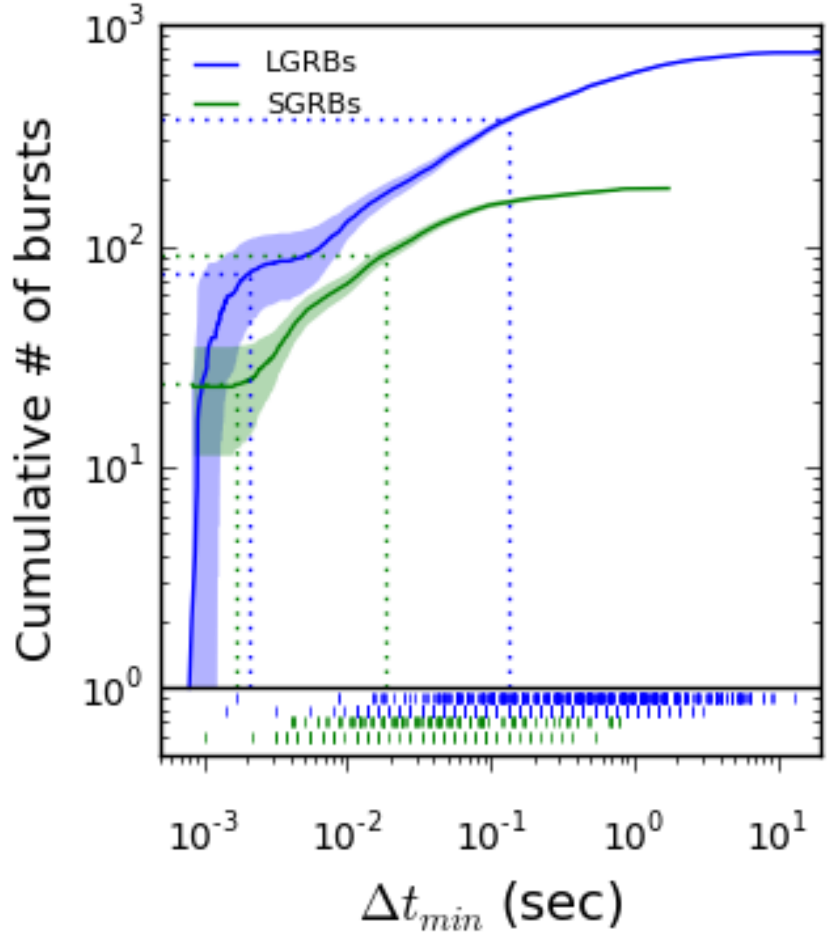
variability time in the proper frame

$$\hat{t}_{\rm var} \sim a_S/c$$

$$t_{\rm var} = (1+z_S)\left(1-\frac{v}{c}\cos\theta_{\rm obs}\right)\varGamma\,\hat{t}_{\rm var}$$

$$a_S \simeq \frac{10^{11}\,{\rm cm}}{1+z_S} \times \left(\frac{t_{\rm var}}{0.03\,{\rm sec}}\right)\left(\frac{\varGamma}{100}\right)$$

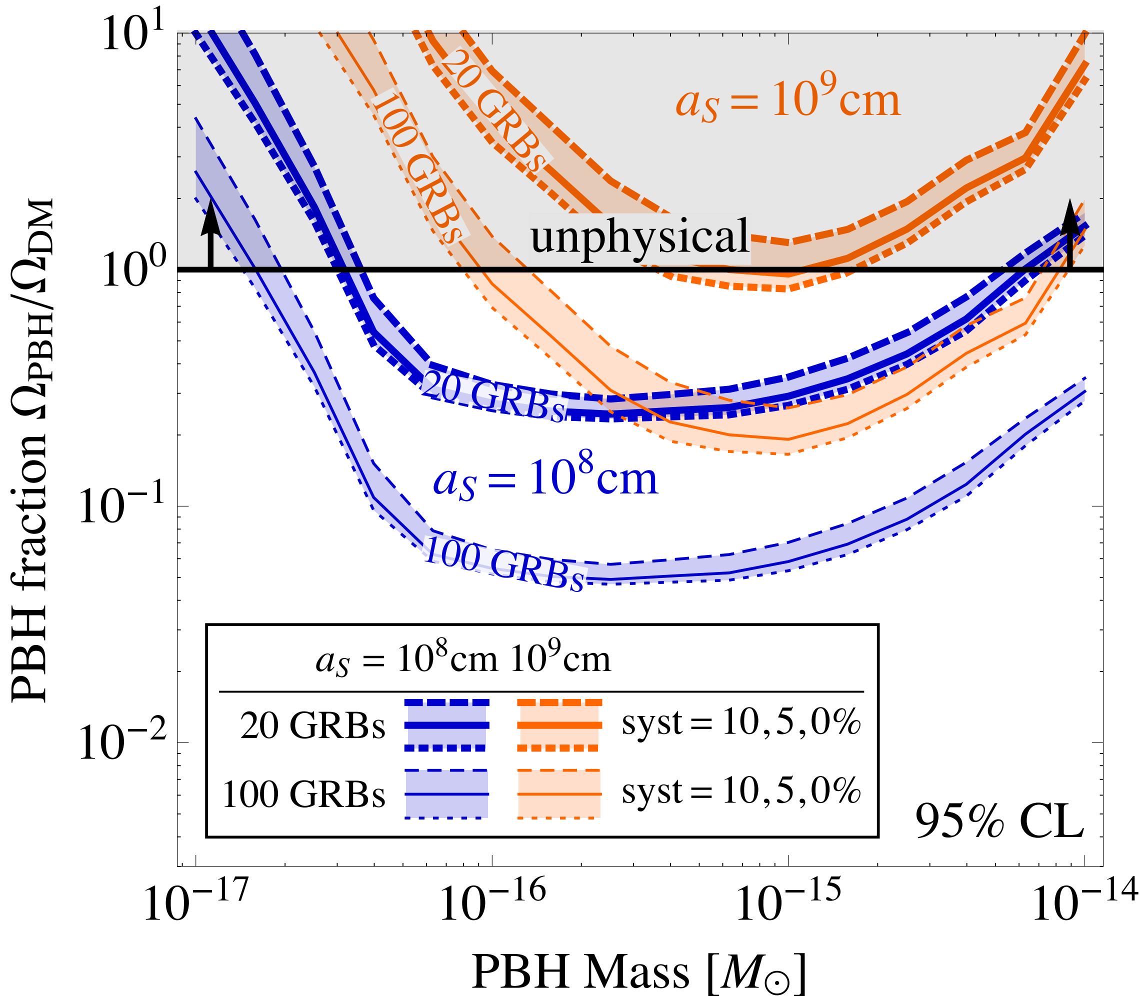
Variabilities of GRBs



Golkhou et al. (2015)

- majority of GRBs have $a_S \sim 10^{11}\mathrm{cm} \gg r_E \sim 10^9\mathrm{cm}$
- however, 10% have $a_S \sim 10^{10}\mathrm{cm}$. Is it possible that there are some with $a_S \sim 10^9\mathrm{cm}$?

Constraints in future ?



We need to better understand GRB sizes…

Application to FRBs

Zheng et al. (2014)
Eichler (2017)

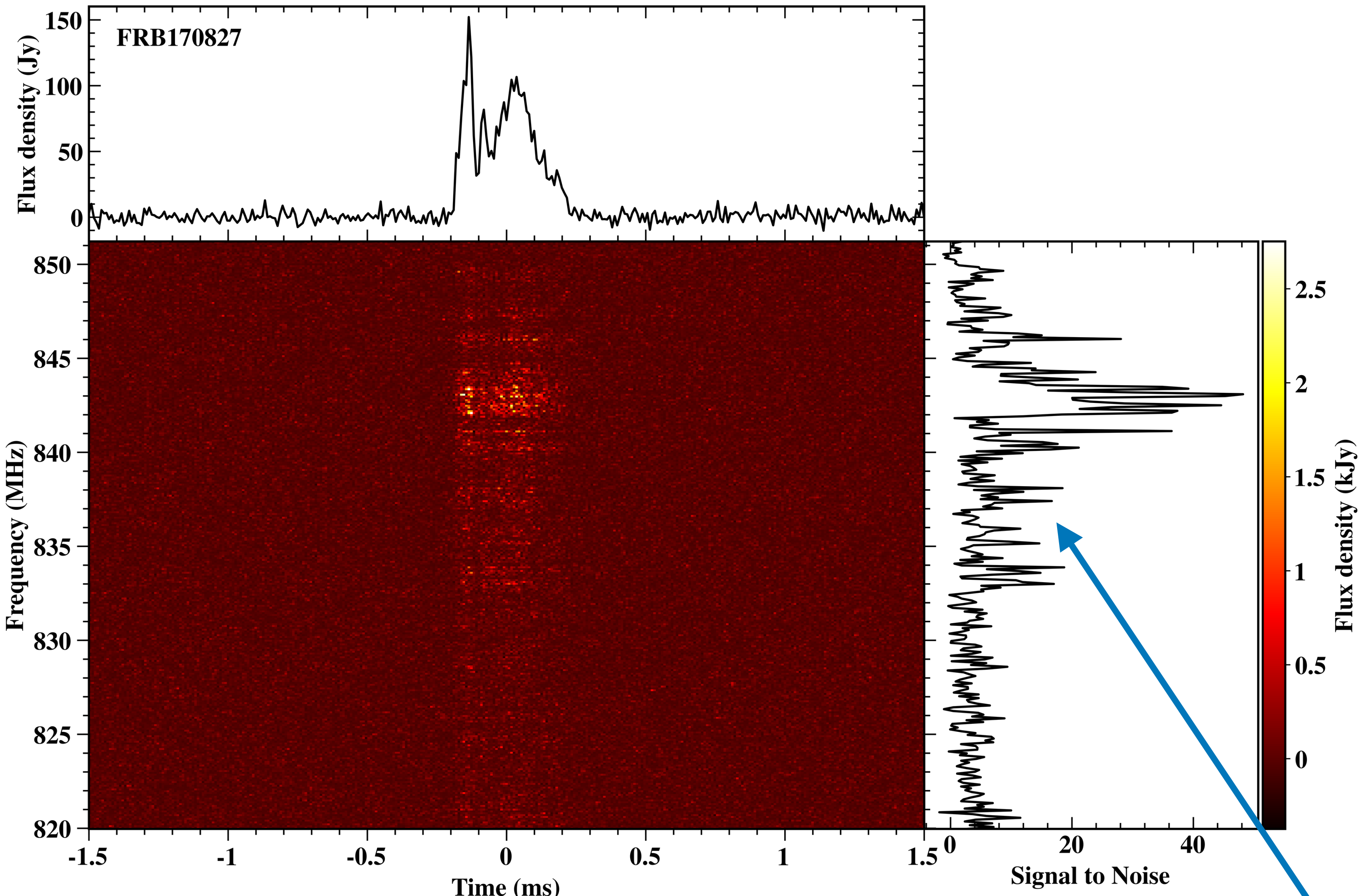
Duration ~ ms

Typically at 400 MHz to 1.5 GHz, but also up to ~ 10 GHz

$\nu \Delta t \sim 1$ ⇒ potentially sensitive to $10^{-4} M_{\odot} < M < 0.1 M_{\odot}$

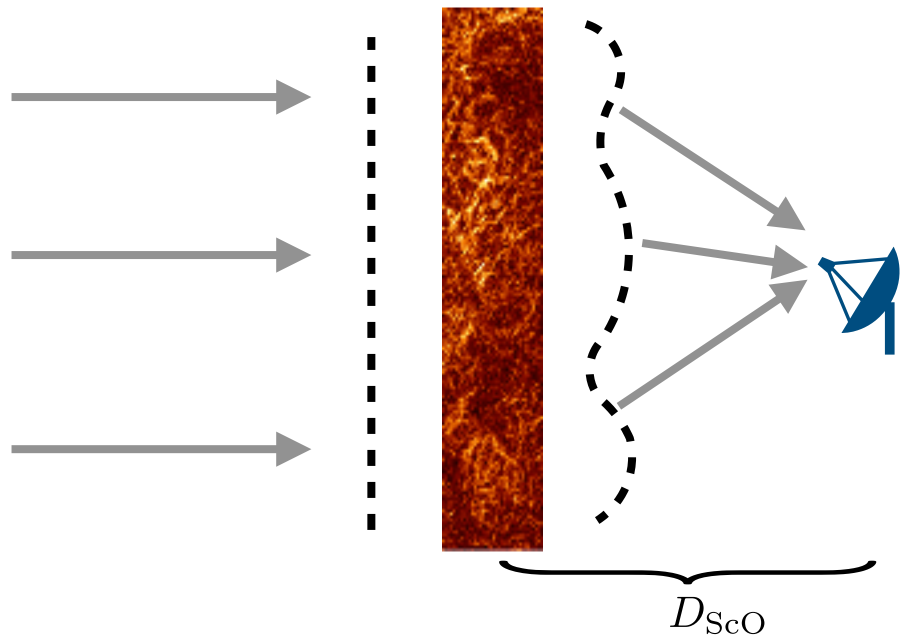
No problems with the size of the source (Einstein radius $10^{-13}\mathrm{cm}$)

Farah et al. (2018)



This is **not** lensing

Scintillation



density of free electrons, inhomogeneous due to turbulence

dispersion of radio waves in plasma: $k = \omega\sqrt{1 - \frac{4\pi e^2 n_e}{m_e \omega^2}}$

$$f_{\mathrm{obs}}(\omega) = \frac{\omega f_{\mathrm{in}}(\omega)}{2\pi i D_{\mathrm{ScO}}} \int d^2x \exp\left\{ \frac{i\omega x^2}{2D_{\mathrm{ScO}}} + i\varphi(\omega, x) \right\}$$

random phase

Scales of scintillation

- phase correlation length $r_{\rm diff}$: $\langle(\varphi(x+r_{\rm diff})-\varphi(x))^2\rangle = 1$

$r_{\rm diff} \propto \omega^{6/5}$ for Kolmogorov spectrum of density inhomogeneities

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$r_{\mathrm{diff}} < r_F \equiv \sqrt{D_{\mathrm{ScO}}/\omega}$ $\Longrightarrow$ strong scintillation

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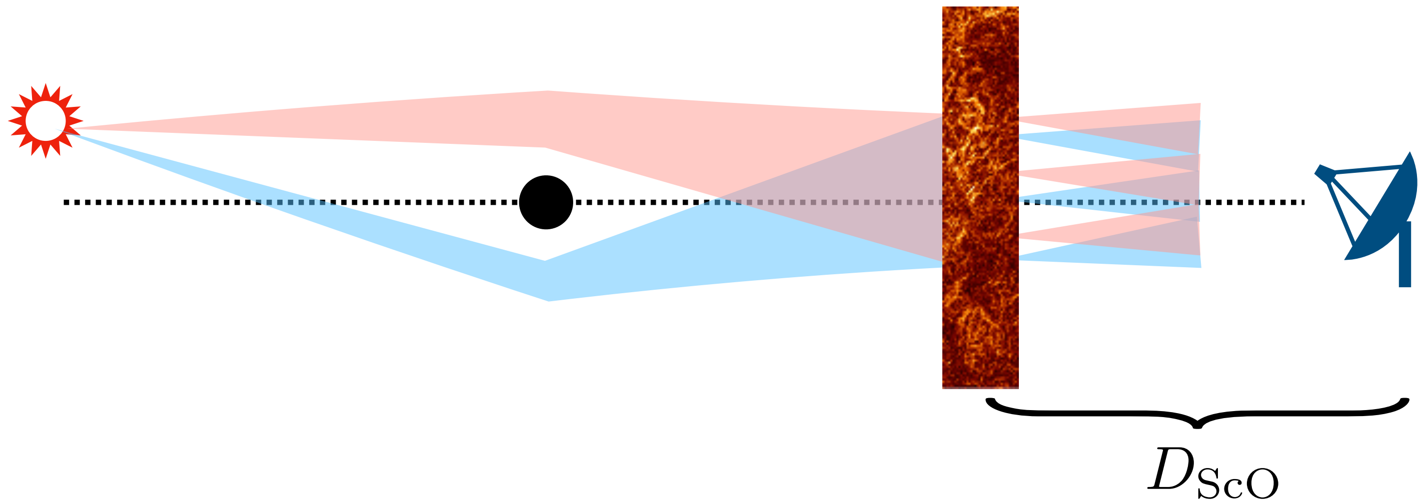
- the light is collected from regions of size $r_{\rm ref} = \theta_{\rm diff} D_{\rm ScO}$

  includes many incorrelated patches

Disentangling lensing from scintillation



$$f_{\rm obs}(\omega) = \frac{\omega f_{\rm in}(\omega)}{2\pi i D_{\rm ScO}} \int d^2x\, e^{i\Phi(\omega,\vec{x})} \left(A e^{i\omega\delta t_A(\vec{x})} + B e^{i\omega[\Delta t + \delta t_B(\vec{x})]}\right)$$

variation of time-delay over the screen

$$|f_{\rm obs}(\omega)|^2 = |f_{\rm in}(\omega)|^2 \left(|\mathcal{A}(\omega)|^2 + |\mathcal{B}(\omega)|^2 + \mathcal{A}(\omega)\mathcal{B}^*(\omega)\, e^{i\omega\Delta t} + \mathcal{A}^*(\omega)\mathcal{B}(\omega)\, e^{-i\omega\Delta t}\right)$$

peak in the FT of intensity spectrum if the amplitude has a slowly varying component

$$\mathcal{A}(\omega) = \frac{A\,\omega}{2\pi i D_{\rm ScO}} \int d^2x\, e^{i(\Phi(\omega,\vec{x}) + \omega\delta t_A(\vec{x}))}$$

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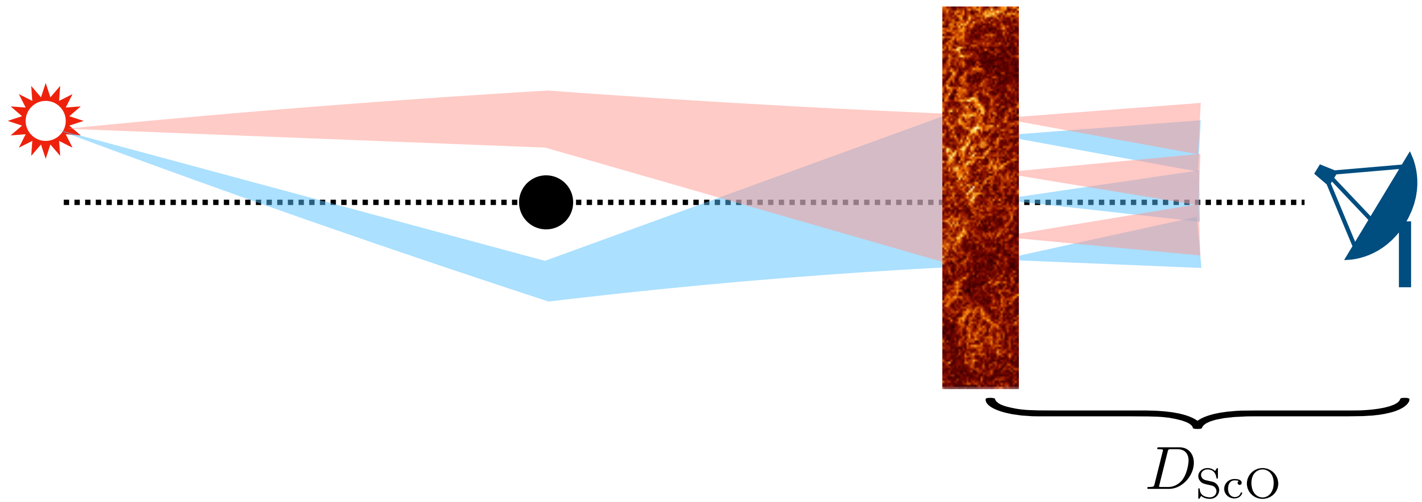

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Smear the amplutude over $\Delta\omega \gtrsim (\Delta t)^{-1}$, square and ensemble average

$$\langle \mathcal{A}(\omega)\mathcal{B}^*(\omega)\mathcal{A}^*(\omega')\mathcal{B}(\omega')\rangle$$
$$\simeq \langle \mathcal{A}(\omega)\mathcal{A}^*(\omega')\rangle \; \langle \mathcal{B}^*(\omega)\mathcal{B}(\omega')\rangle + \langle \mathcal{A}(\omega)\mathcal{B}^*(\omega)\rangle \; \langle \mathcal{A}^*(\omega')\mathcal{B}(\omega')\rangle$$

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$$\begin{aligned}
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\end{aligned}$$

$$\Delta\omega < \omega_{\rm dec} = \frac{2D_{\rm ScO}}{r_{\rm ref}^2}$$

(the random phases do not change, have smooth frequency dependence)

Smear the amplutude over $\Delta\omega \gtrsim (\Delta t)^{-1}$, square and ensemble average

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contributes order one if

$$\Delta\omega < \omega_{\rm dec} = \frac{2D_{\rm ScO}}{r_{\rm ref}^2}$$

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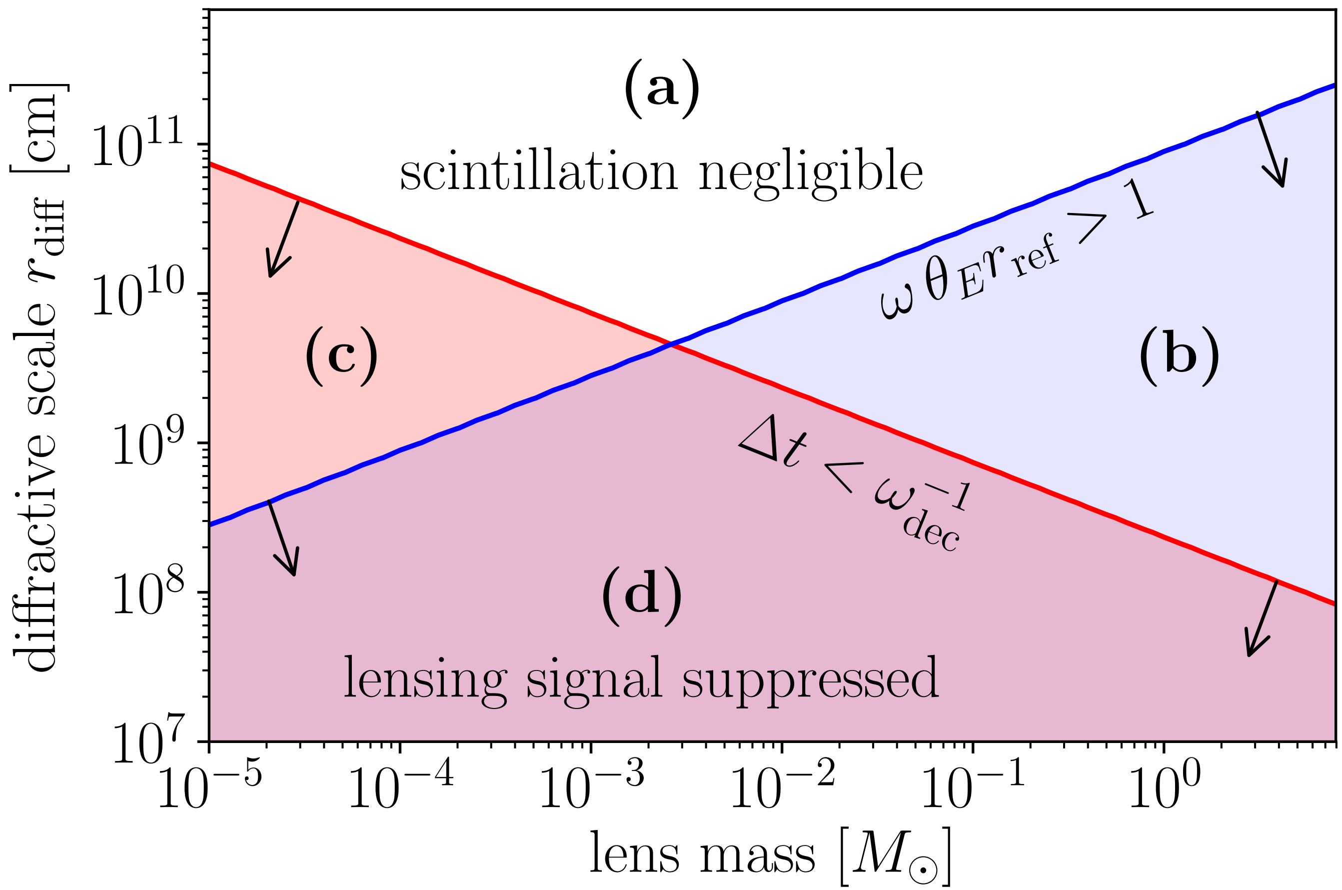
contributes order one if

$$\omega\theta_E r_{\rm ref} < 1$$

(the images are distorted in the same way,
“are not resolved by the screen”)

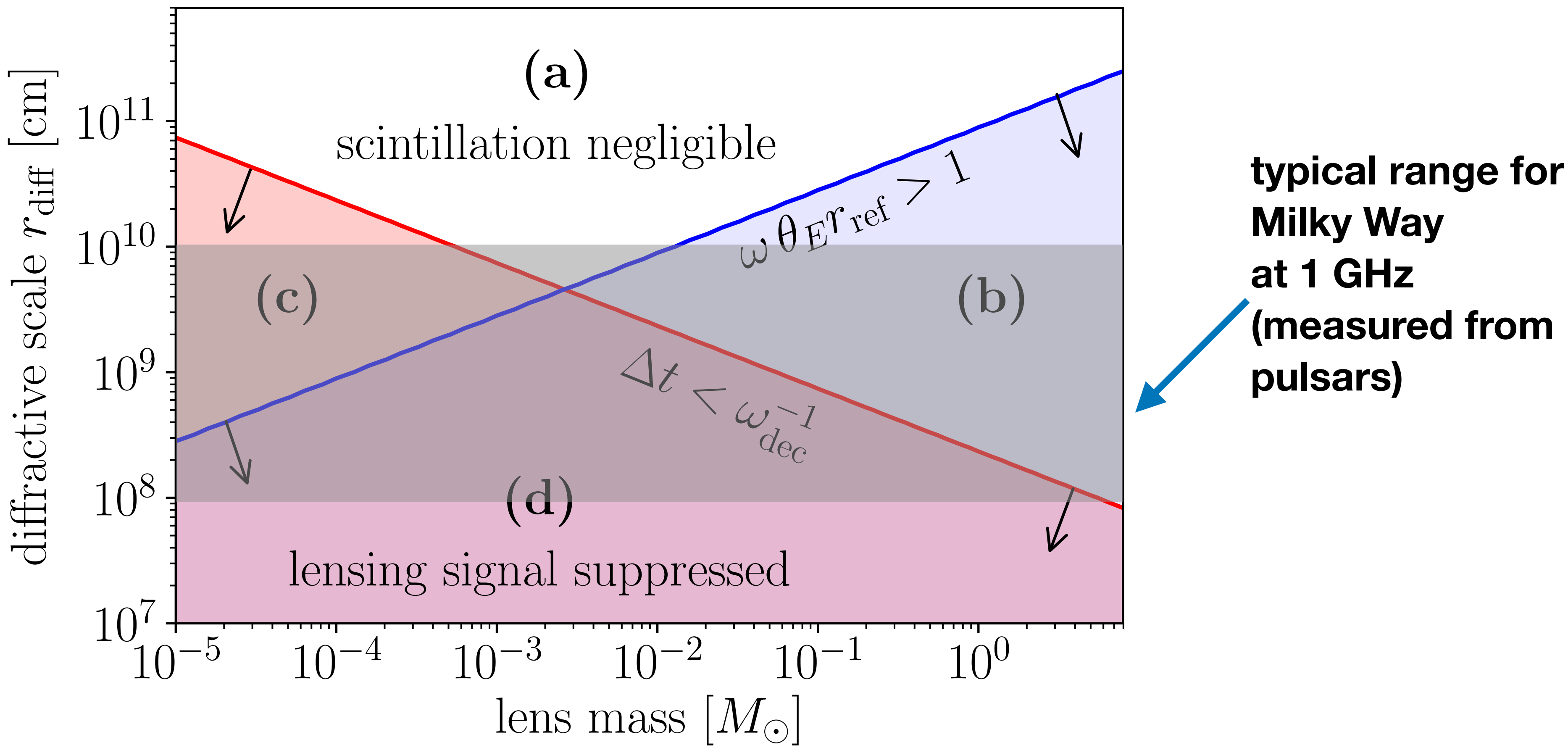


(a) $\Delta t \gg (\omega_{\rm dec})^{-1},\ \ 1 \gg \omega\,\theta_E\,r_{\rm ref}$

(b) $\Delta t \gg (\omega_{\rm dec})^{-1},\ \ \omega\,\theta_E\,r_{\rm ref} \gg 1$

(c) $(\omega_{\rm dec})^{-1} \gg \Delta t,\ \ 1 \gg \omega\,\theta_E\,r_{\rm ref}$

(d) $(\omega_{\rm dec})^{-1} \gg \Delta t,\ \ \omega\,\theta_E\,r_{\rm ref} \gg 1$



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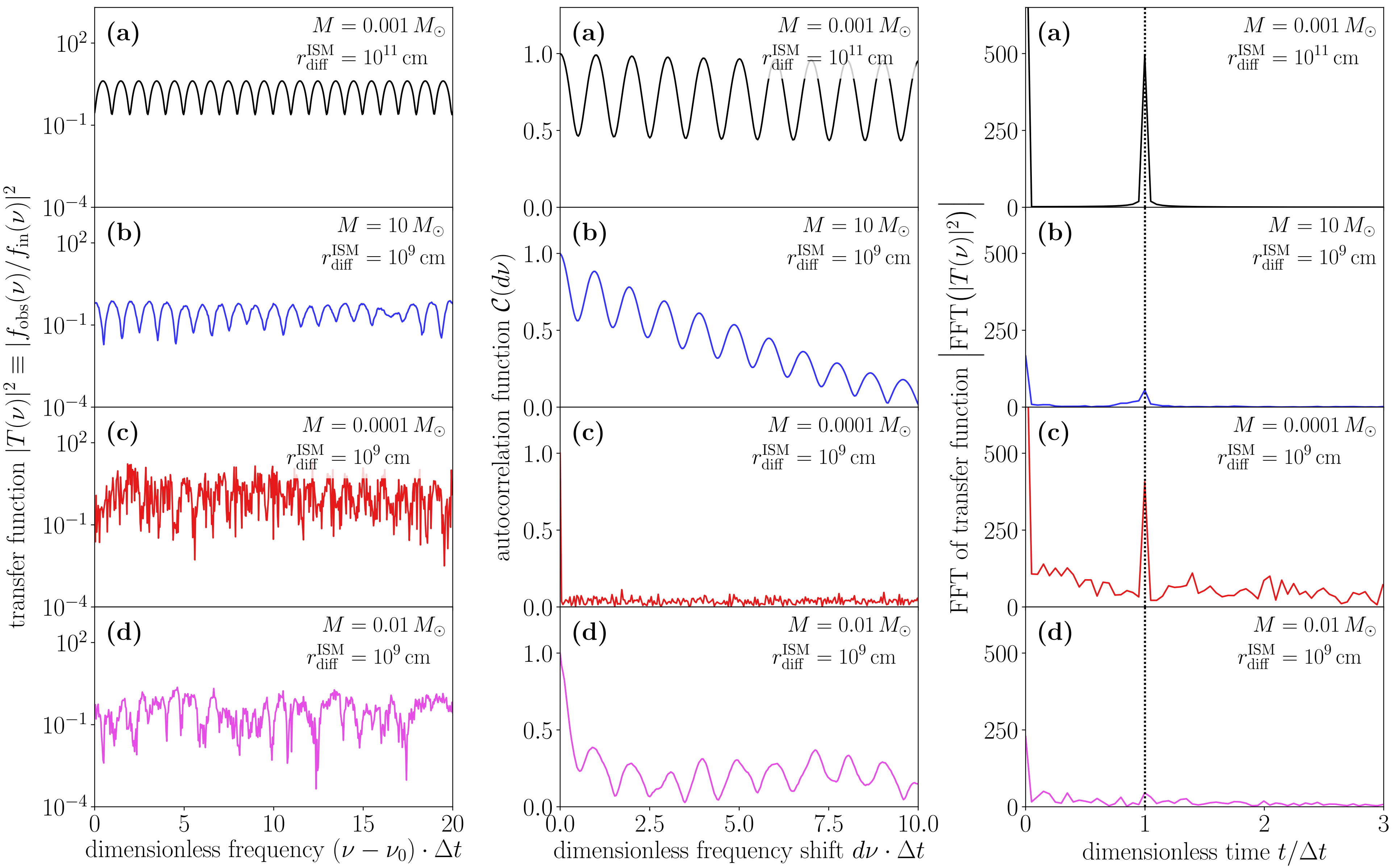
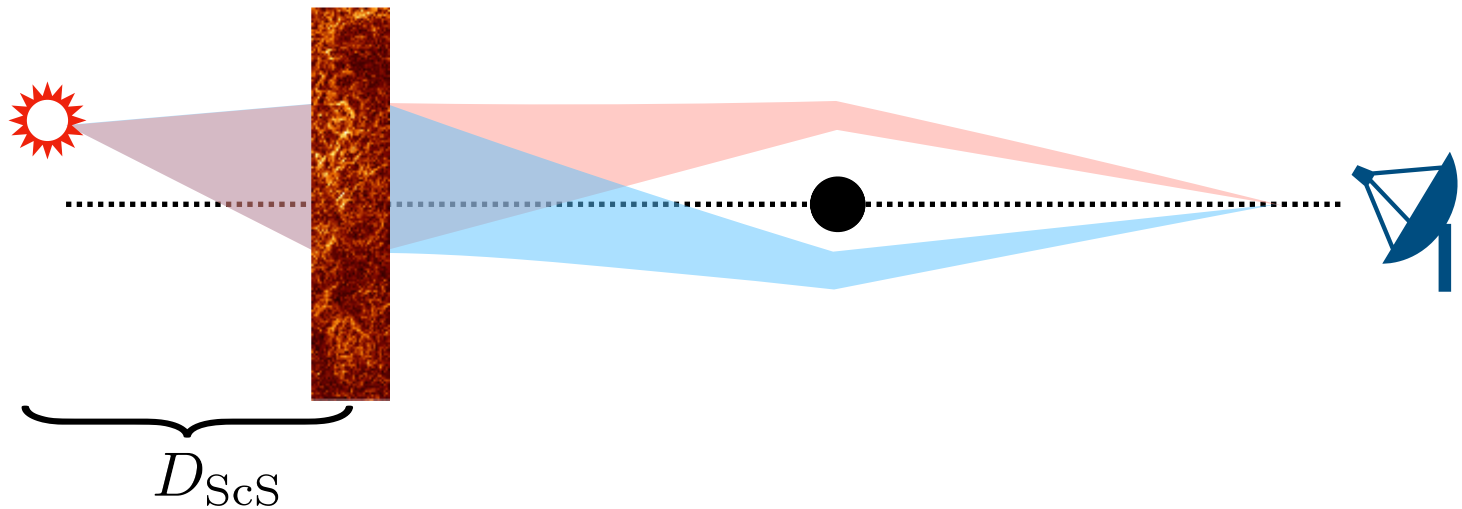
Simulating lensing+scintillation



Scintillation outside MW

- in the host galaxy: symmetric, the same analysis as for MW

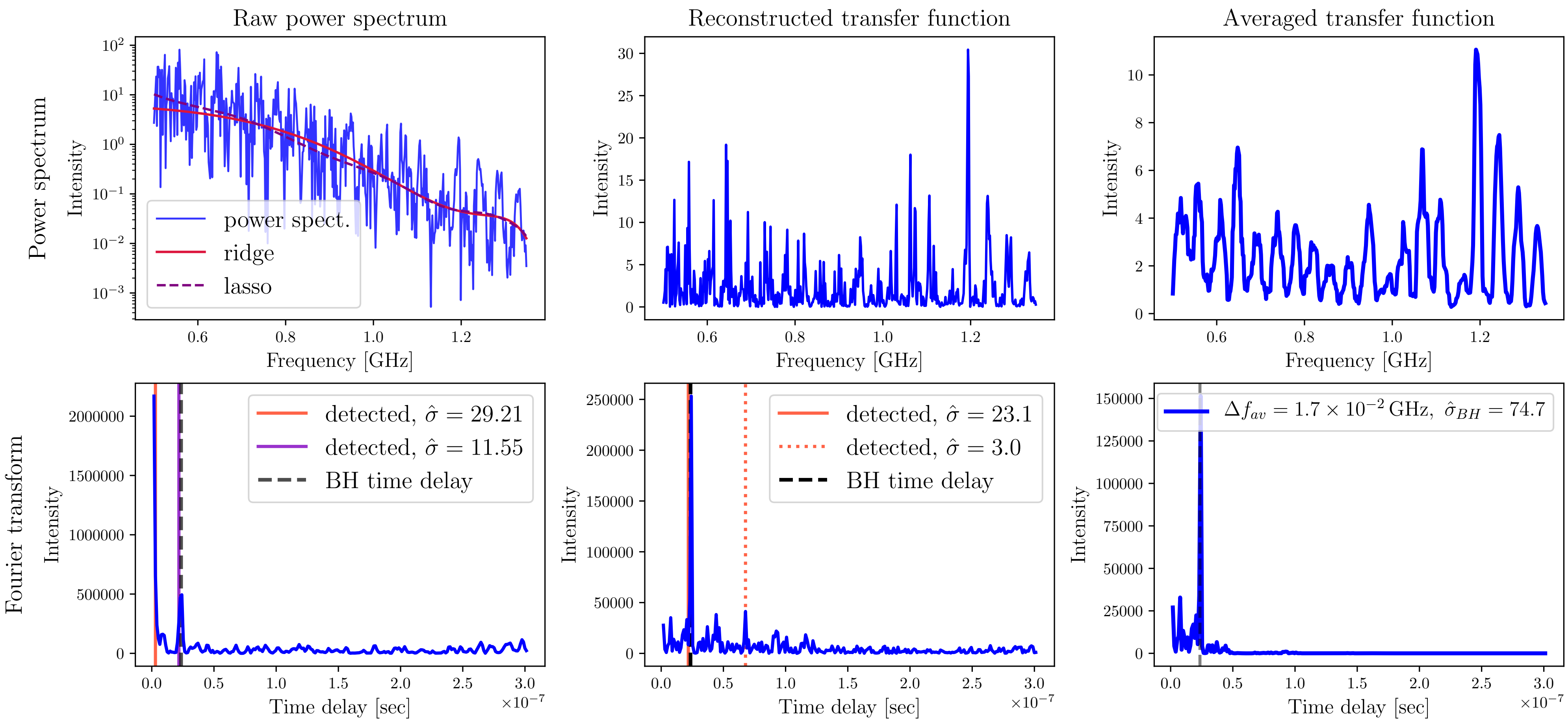


- in the intergalactic medium: unimportant for realistic assumtions, unless the line-of-sight crosses a galaxy or a cluster

Analysis of simulated data

Idea: Apply peak-finder algorithm to the FFT of the power spectrum

$$M = 0.001\,M_\odot, \quad r_{diff} = 10^9\,\mathrm{cm}, \quad y = 0.5, \quad D_L = 0.5\ \mathrm{Gpc}$$



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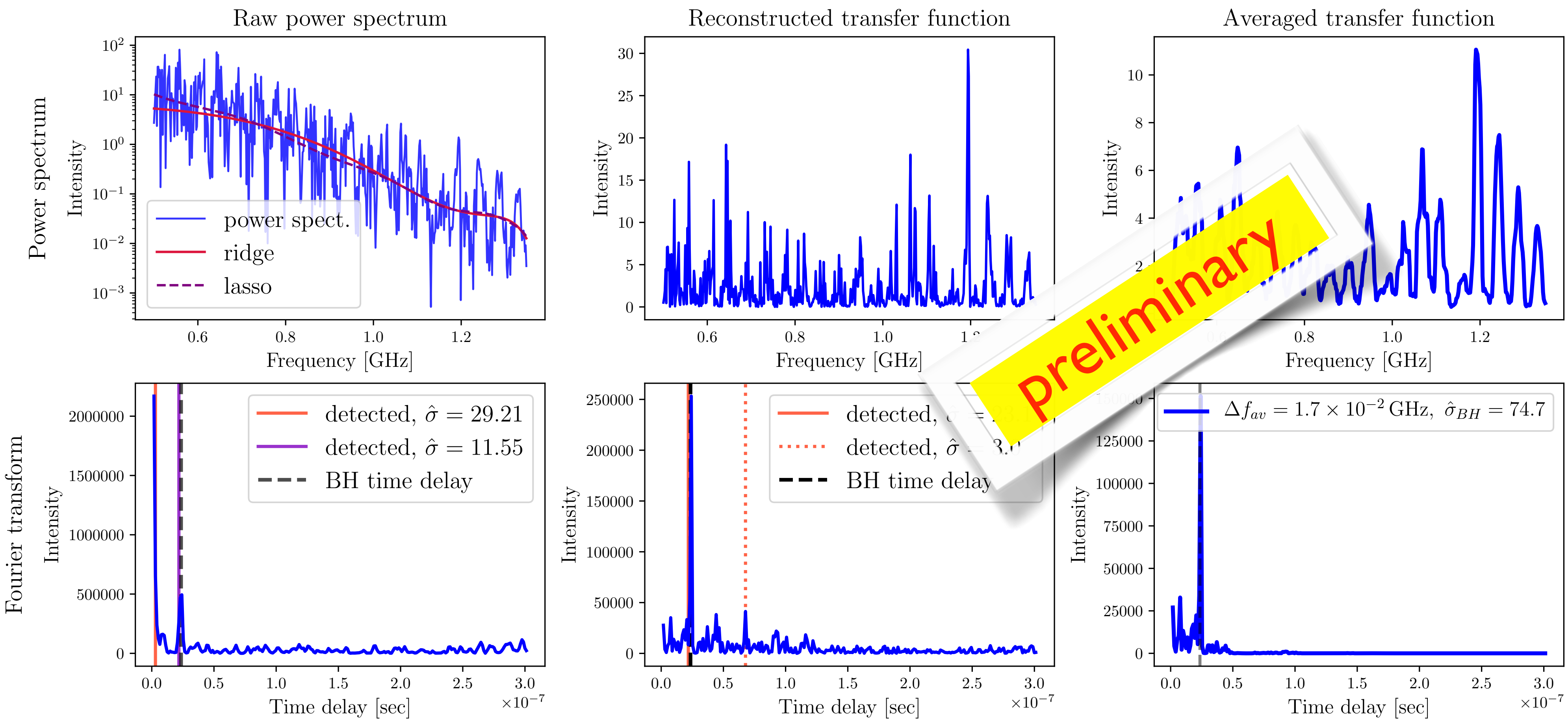

Future reach